The Road Not Taken - What Is The "Appropriate" Path to Development When Growth is Unbalanced?*

Ahmed S. Rahman
Department of Economics
United States Naval Academy
Annapolis, Maryland

June 2009

Abstract

This paper models a developing economy that endogenizes both directed technologies and demography. Potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction allows us to evaluate potentially divergent development paths. I find that exogenous unskilled-labor biased technological growth can induce higher fertility and lower education, inhibiting overall growth in per person income. However, if technical progress is *locally endogenous*, the growth in the overall workforce caused by unskilled intensive technological progress can make R&D more profitable; this can actually induce more income growth can the alternative, skill-intensive path.

- *Keywords*: directed technical change, demography, human capital, fertility
- *JEL Codes*: O31, O33, J13, J24

*Many thanks to Eran Binenbaum, Gregory Clark, Robert C. Feenstra, Petra Moser, Giovanni Peri, Katheryn N. Russ, Alan M. Taylor, Florence Bouvet, and Chad Sparber for helpful suggestions. All errors are my own.

1 Introduction

The last half century has seen the robust growth of some nations and the persistent stagnation of others. This is particulary true of the developing world; while rich nations have maintained fairly consistent rates of growth (2 or 3% per annum), poorer nations have traversed widely different growth paths (between -1 and 7%). This paper suggests a possible reason behind such divergence by producing a model emphasizing the interdependence between directed technical change and demography. In this model, potential innovators decide which technologies to develop after considering available factors of production, and individuals decide the quality and quantity of their children after considering available technologies. This interaction creates the possibility for multiple growth paths - some economies develop labor-intensive techniques and expand the pool of unskilled labor; others grow into societies of highly skilled individuals using sophisticated skill-intensive techniques. Which path will lead to greater prosperity is the primary focus of this paper.

The model emphasizes how economic growth can often be an unbalanced process involving critical production choices. A farm can be maintained either with uneducated farmers wielding hand tools, or with farmers skilled in using agronomic instruments and automated machinery. A factory can be structured as an assembly line run mainly with unskilled workers supervised by a few skilled ones, or as a computer-controlled facility mainly run by skilled workers with a few unskilled janitors.[1] A road can be built using lots of manual labor physically laying down stone and brick by hand, or construction workers trained in using bulldozers and steamrollers. These examples highlight not only that technologies can be directed towards particular factors, but also that each country can take its own unique development path, producing similar things in very different ways.

This paper boils down all these considerations into a simple question - would the macro economy be better off on a skilled-intensive path or an unskilled-intensive path? The answer for each country, of course, is that it depends. It depends on how *productive* skilled and unskilled labor already are. It depends on how *abundant* skilled and unskilled labor are. And it depends on how technological changes can affect future supplies of skilled and unskilled labor.

By exploring the simultaneity between technological changes directed towards particular factors and the factors themselves, we can explore some of these issues. This approach constitutes a notable departure from the existing literature on technologies that augment specific factors or sectors.[2] These works often highlight the "inappropriateness" of growth in technologies that can be implemented by only a small portion of the economy. For example, Basu and Weil (1998) and Acemoglu and Zilibotti (2001) illustrate how technologies designed for capital-intensive (physical

[1] This example comes from Caselli and Coleman (2006)

[2] Katz and Murphy 1992; Acemoglu 1998; Kiley 1999; Acemoglu and Zilibotti 2001; Xu 2001; Acemoglu 2002, to name but a few papers.

or human) societies that diffuse to developing regions are used ineffectually there, if at all. And Mokyr (1999) explains that the British Industrial Revolution initially produced only minor improvements in living standards because technical progress occurred in just a few small industries. These papers suggest that technologies catered for the abundant factors of production are more appropriate for the economy and will provide robust future growth. Thus poorer, labor abundant countries should develop labor-intensive technologies to make their large uneducated workforce more productive.

But these works typically do not take into account that these factors can themselves evolve, and will adjust to changing economic circumstances.[3] If factors do change in these models, they typically do so exogenously, in order to explore the subsequent technological changes that can occur. But this partial equilibrium approach may mislead us in some ways, particulary when it comes to *long-run* growth. Allowing for the co-evolution of factors and technologies can alter our perspective of the "appropriate" technological path. Two new considerations emerge with this approach. The first is that if unbalanced growth also promotes growth in the more productive factor, overall growth will be higher than the alternative path. The other consideration is that different technological paths can produce different rates of population growth. All else equal, the path with the lower population growth rate will produce more income per person.[4]

With simulations of the model, we discover a number of things. First, exogenous unskilled-labor biased technological growth can induce higher fertility and lower education, inhibiting overall growth in per person income. This case highlights that, contrary to the appropriate technology literature, the overall size of the factor or sector may not be the most important consideration, for the dynamic effects of technological change geared towards such factors or sectors may work against long-run prosperity. However, if technical progress is *locally endogenous*, the growth in the overall workforce caused by unskilled intensive technological progress can make R&D more profitable; this can actually induce more income growth can the alternative, skill-intensive path. Thus we see that the *source* of technological growth may be very important in answering our titular question - skill-biased technologies can indeed be appropriate for development if they exogenously flow from other economies like manna, even if skilled labor is in relative short supply. On the other hand, unskill-biased technologies can be appropriate for development if they are home-grown, even if it produces some ostensibly negative side effects like population growth and a de-skilled workforce.

We should note that this paper heavily borrows from Acemoglu's important work on directed technological change (Acemoglu 1998, 2002). But this work departs from that literature in two

[3]Papers that do consider interactions between technology and human capital include Stokey 1988, Chari and Hopenhayn 1991, Grossman and Helpmann 1991, Young 1993, Redding 1996, Galor and Weil 2000, and Galor and Moav 2000. None however assess the appropriate path to development for an economy in the context of such simultaneity.

[4]Galor and Mountford (2006) stress this point in explaining divergent growth paths in history.

fundamental ways. First, the literature relies almost exclusively on analyzing *balanced* growth paths, while I look solely on the *unbalanced* case, implicitly assuming that countries always face a choice in its overall growth direction. Second, as already mentioned, the literature also almost always treats factors of production as exogenously determined,[5] whereas here they are endogenous in the model.

The rest of the paper is organized as follows. Section 2 motivates the paper by looking at some cross-country data. Section 3 presents the full model in steps, first presenting a model of endogenous technological bias, and then merging this with a simple theory of demography. This model then motivates our simulation experiments in section 4. Section 5 provides some concluding remarks.

2 Some Data

2.1 A Cross-Section of Factor-Specific Technologies

We begin by taking account of estimated factor-specific productivities of a cross-section of countries. Consider the following production function for country i:

$$Y = \left[(A_{l,i}L_i)^{\sigma} + (A_{h,i}H_i)^{\sigma}\right]^{1/\sigma} \tag{1}$$

Here we specify production as one with a constant elasticity of substitution between skilled and unskilled labor aggregates (this elasticity being $1/(1-\sigma)$. $A_{l,i}$ is the efficiency of unskilled labor in country i and $A_{H,i}$ is the efficiency of skilled labor in country i.[6]

Furthermore, if factors of production are paid their marginal products, the "skill-premium" may be written as:

$$\frac{w_{h,i}}{w_{l,i}} = \left(\frac{A_{H,i}}{A_{L,i}}\right)^{\sigma}\left(\frac{H_i}{L_i}\right)^{\sigma-1} \tag{2}$$

Caselli and Coleman (2006) note that one can study cross-country productivity differences using equations (1) and (2), for these simply represent two equations with two unknowns. That is, given data on Y_i, L_i, H_i, and $\frac{w_i^h}{w_i^l}$, we can back out each country's implied pair of technological coefficients and compare them.[7]

[5] Acemoglu 1998 relegates the possibility of endogenously determined human capital in the appendix to his paper, while he does not discuss the possibility either in Acemoglu 2002 or in the chapter on directed technical change in his recent growth textbook (Acemoglu 2008).

[6] This functional form resembles the production function used in section 3, where we endogenize technological growth; efficiency coefficients will proxy for the breadth and depth of factor-complementary machines.

[7] The data is also from Caselli and Coleman (2006). Y is average GDP per capita for 1985-1990, taken from the Penn World Tables. Labor levels are constructed using the implied Mincerian coefficients from Bils and Klenow

Key to this exercise is our parameter choice for $\sigma \leq 1$. Careful empirical labor studies such as Autor et al (1998) and Ciccone and Peri (2005) have found that the elasticity of factoral substitution between more and less skilled workers most likely lies between 1 and 2.5 (consistent with a value of σ between 0 and 0.6). Both for this exercise and the simulations in section 4, we choose a benchmark value of $\sigma = 0.5$ for a proxy elasticity parameter most applicable for a wide range of countries and for a wide variety of skill-unskilled categories.[8]

Table 1 reports these backed out measures of A_l-A_h pairs. Figure 1 depicts the relationships between relative technical skill-bias (A_h/A_l), relative skill-endowments (H/L), and income per capita across a broad array of countries. Immediately clear is the positive associations between technical skill-bias and skill endowment, and between technical skill-bias and income levels. These positive relationships hold whether we consider a skilled worker as someone who completed primary school, or someone who completed high school, or even someone who completed college. This was precisely one of the main points behind Caselli and Coleman's study. Not only do wealthy nations enjoy large pools of human capital, but they also employ this capital far more effectively than poorer nations.

But, from these static pictures it is not immediately clear which technological path would produce greater results for any particular country over time. On the one hand, a country with a relative abundance of unskilled labor should greatly benefit by making them more productive. On the other hand, unskilled labor's *level* of productivity may already be fairly low; unskilled-bias technical change that induces a rise in L and a fall in H would then lower the relatively-more productive factor and raise the relatively-less productive factor.

So, informative as these scatterplots are, they bring up many questions. One is the standard chicken-or-egg question. Does a country naturally endowed with a lot of human capital imitate/develop technologies best suited for this type of skilled workforce, becoming wealthy in the process? Or rather, does a country blessed for whatever reason with a rich pool of skill-intensive knowledge inherently provide the incentives necessary for growth in education? That is, is the path to wealth a technology story, or an investment story?

Second but no less a puzzle is the question of whether *un*-skilled-intensive productivity can produce macroeconomic prosperity on average. According to standard directed technical change theories (Kiley 1999, and Acemoglu 2002), if a country is populated primarily by the un- or under-educated, local innovators would simply direct their innovative energies toward technologies that would complement them (profiting from so-called "market-size" effects). Figure 1 suggests that this indeed happens - the more relatively abundant is unskilled labor, the greater it's relative productivity. But whether this can produce more output than an alternative skill-intensive

(2000). Wages for skilled and unskilled are constructed using Mincerian coefficients and the duration in years of the various schooling levels. See their paper for more details.

[8]Ciconne and Peri (2005) themselves estimate σ to be 0.5 when considering U.S. high school dropouts as unskilled labor and high school graduates as skilled labor (although their preferred measure is 0.33).

approach remains unclear.

We begin exploring some of these issues by allowing the factors of production to *respond* to biased technological changes, first in a comparative static experiment in section 2.2, and then in a fully specified general equilibrium model in section 3.

2.2 Unbalanced Growth - A Comparative Static Experiment

Here we consider changes in output, Y, that can occur when we have the factors of production respond to exogenous unbalanced technological growth. First, let us totally differentiate the production function given by (1):

$$dY = \left(\frac{\partial Y}{\partial A_l}\right) dA_l + \left(\frac{\partial Y}{\partial A_h}\right) dA_h + \left(\frac{\partial Y}{\partial L}\right) dL + \left(\frac{\partial Y}{\partial H}\right) dH \tag{3}$$

Both types of technologies and both types of factors have the potential to change. Let us assume that when technologies are *biased* towards factor L, it induces L to rise and H to fall (higher unskilled-intensive productivity makes some people become unskilled laborers instead of skilled ones). On the other hand, technological growth that is biased towards H induces L to fall and H to rise (higher skilled-intensive productivity makes some people become skilled laborers instead of unskilled ones). That is, $dA_l > 0 \Rightarrow dL > 0$ and $dH < 0$. And $dA_h > 0 \Rightarrow dL < 0$ and $dH > 0$.[9] Let us consider two possibilities. The first is where $dA_l = 1$ and $dA_h = 0$. This is the case of unskilled-bias technological change (where the total change in output can be written as dy_{unsk}). The second case is where $dA_h = 1$ and $dA_l = 0$. This is the case of skill-bias technological change (where the total change in output can be written as dy_{sk}). Finally, let us call a factoral change dF, and assume for convenience that factors respond linearly and in exactly opposing ways. That is, with $dA_l = 1$, $dL = -dH = dF$, and with $dA_h = 1$, $-dL = dH = dF$.[10]

When there is unskill-biased technological change, the total change in income per capita can be written as

$$dY_{unsk} =$$

$$[(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (L \cdot dA_l + A_l \cdot dL) + (A_h H)^{\sigma-1} (A_h \cdot (-dH))\right) \tag{4}$$

where dY_{unsk} is the total change in income per capita with unskilled intensive growth. Here we assume that A_h does not change (hence $dA_h = 0$) and that this type of technological growth has

[9] Note that only when $\sigma > 0$ can we consider A_l unskilled *biased* and A_h skilled *biased*. This is a reasonable assumption given previous estimates of σ. See Acemoglu 2002 for a fuller discussion.

[10] This is only to make the comparison between the two growth paths simple - it is not a critical assumption for the basic story.

de-skilling effects (hence the negative sign in front of dH). On the other hand, when there is skill-biased technological change, the total change in income per capita can be written as

$$dY_{sk} =$$

$$[(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} \cdot \left((A_l L)^{\sigma-1} (A_l \cdot (-dL)) + (A_h H)^{\sigma-1} (A_h \cdot dA_h + (A_h dH))\right) \quad (5)$$

where dy_{sk} is the total change in income per capita with skilled intensive growth. Here we assume that A_l does not change (hence $dA_l = 0$) and that this type of technological growth induces erstwhile unskilled individuals to become skilled (hence the negative sign in front of dL).

We want to know whether or not unbalanced unskill-intensive growth is *better* than unbalanced skill-intensive growth (once we account for factoral responses to such changes). That is, we ask if, for a certain value of dF, whether or not the condition $dy_{unsk} > dy_{sk}$ holds? Playing around with different values of dF for particular countries, we come to a couple of propositions:

Proposition 1 *If factors do not respond to technological changes, unskilled-bias technological change produces more output than skill-biased technological change.*

Proposition 2 *The more responsive are factors to biased technological changes, the greater are the relative output gains from skill-biased technological change.*

At first blush the first proposition may be surprising. Wealthy countries, having lots of H, should greatly benefit from increases in A_h. But, given Caselli and Coleman (2006)'s categorization and calculations of L and H, L tends to be the abundant factor even for wealthy nations. That is, the number of workers categorized as unskilled are greater than workers categorized as skilled, even for those countries with relatively large endowments of skilled labor. And since factors are grossly substitutable, technologies used by the more abundant factor will generate the greater aggregate gain.

The second proposition simply follows from the fact that H is inherently the more productive factor. This comes both from its relative scarcity (so its marginal productivity tends to be higher even if technologies are symmetrical) and from the higher productivity coefficient on H compared to the one for L. So if labor tends to readily switch from one type to the other with unbalanced technical progress, skill-intensive growth tends to produce more output.

Combining both observations, we see that each country has a *threshold level* of factoral responsiveness (call it dF^*), whereby $dY_{sk} = dY_{unsk}$.

Figures 2 illustrate a few country-specific cases. For three countries (Argentina, China and the U.K.) we plot $dY_{sk} - dY_{unsk}$ against dF. Highlighted lines are where factor responsiveness is high enough whereby skill-bias technical change produces more output than unskill-bias technical change. So for example, if we consider H to be those with at least a high-school degree, then an increase in A_h by one unit must induce an increase in H and a decrease in L by at least 0.39 units for this change to generate more GDP than an equal increase in A_l in Argentina. In Great Britain that same one unit increase in A_h must induce an increase in H and a decrease in L by at least 0.25 units for the change to generate more GDP than an equal increase in A_l.

Thus we see that because countries have their own unique pairs of factor supplies and productivities, they will have different dF^*s. Figure 3 plots each country's threshold level of factor responsiveness (dF^*, the dF where $dY_{sk} = dY_{unsk}$) against its GDP per capita. We immediately see that the poorer the nation, the greater will factors need to respond to technological changes for skill-intensive growth to be the superior path to development. For example, suppose that when H is categorized as those with a primary school education or more, $dF = 2$ for the whole world. Then, we can say that countries like China, Ghana, India, Honduras, Kenya, and the Philippines would be better off with growth in A_l, while most other countries would be better off with growth in A_h.

Interestingly, as we compare the top and bottom scatterplots we can see that the more narrow is our definition of H, the *smaller* is the threshold factor responsiveness. This is simply because increases from the relatively more scarce factor produces greater benefits, for the marginal productivities of the more scarce factor tends to be larger. This in effect flips Acemoglu's discussion of market-size effects on its head: if factors are allowed to respond to technological change, such change that augments the *less* abundant factor may produce *more* output in the longer run.

Thus we need to better understand how elastic these factors of production are in the context of directed technical change. On top of this, we should acknowledge that factors probably do not respond to technological changes in such linear and symmetrical ways, as implied above. And finally, factor changes may themselves lead to subsequent changes in biased technologies. So we should move beyond this comparative static analysis to a model that endogenizes both factors and technologies in a general equilibrium framework to help us better consider these issues. That is, by actually endogenizing the micro-economic incentives for researchers and families, we can generate actual values for dA_l, dA_h, dL and dH over time.

2.3 A Cross-Section of Demographic Characteristics

One final consideration we should make before launching into a model is that technological changes can also induce changes in overall population levels through fertility shifts. Indeed education and fertility decisions at the household level should be considered as going hand in hand; this is the so-called quality-quantity tradeoff inherent in child-rearing (Becker and Lewis 1973; Becker and Barro 1988).[11] Supporting a quality-quantity tradeoff approach is the observation that in

[11]This is also stressed by Galor and Mountford (2006, 2008). In these papers population changes come from trade specialization patters. Technological changes however can affect demographic patterns in very similar ways.

developing countries educated workers have a much lower birthrate than uneducated workers (Kremer and Chen 2002). So, along with a combination of technologies, a country simultaneously chooses a combination of factors through household decisions on education and procreation. A cross-country sample of these decisions are depicted in Figure 4. Here observations are sized according to the country's income per capita. We can observe a negative correlation between population growth and education, a negative correlation between population growth and living standards, and a positive correlation between education and living standards.

It is also commonly believed that child labor is a symptom of the technological path an economy is on (Edmonds and Pavcnik 2005). This should be important for us as another example of the quality-quantity tradeoff, since a child working is a child not in school. The International Labor Organization's Statistical Information and Monitoring Program on Child Labor (SIMPOC) most recently estimated that 211 million children, or 18% of children 5-14, are economically active worldwide (ILO 2002). The bottom of Figure 4 illustrates the strongly negative relationship between living standards and the percentage of 10-14 year-olds who are employed. It would not be implausible then to suggest that increases in unskilled-intensive technologies may also raise child labor participation rates.

Thus we have another reason to focus on the responsiveness to unbalanced technological change. A high fertility rate and child-labor participation rate are the likely consequences of an economy taking an unskilled-labor intensive technological path, and not necessarily the mere symptoms of these countries' relative poverty. The question we want to use the model to help answer is, would an alternative path, one that limits child labor and checks population growth, be any better?

3 The Model

In this section we present a simple model where technological and demographic variables interact and co-evolve. Subsection 3.1 illustrates the production side of the economy. Subsection 3.2 illustrates the research and technology side. Subsection 3.3 merges this approach with a simple theory of demography; the combined model allows us to evaluate the co-evolution of factors and technologies so that we can judge in the next section the true "appropriateness" of different kinds of unbalanced technological developments.

3.1 Production

Consider a discrete-time economy. We use the production function given by (1) but now we explicitly specify factor-specific technologies. Specifically production is specified as the following.

$$Y = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{1/\sigma} \tag{6}$$

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L} \right)^\alpha dj \qquad A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H} \right)^\alpha dk \tag{7}$$

Here both types of labor (unskilled L, and skilled H) work with intermediate "machines" to produce a homogenous final output. I make the rather stringent but typical assumption that these machines can complement either skilled labor or unskilled labor, but not both. Machines (of type j) which complement unskilled labor are denoted by $x_l(j)$, while machines (of type k) which complement skilled labor are denoted by $x_h(k)$.

The parameter σ indicates the degree of substitutability between skill and unskill-intensive "sectors" in aggregate production. When $\sigma = 1$, the production function is linear; when it is 0, the production function is Cobb-Douglas; when it is $-\infty$, the production function is Leontieff. As mentioned in section 2.1, estimates of this elasticity clearly place σ above zero; thus we will assume that these sectors are grossly substitutable, so *unbalanced* growth (progress that is confined to just one of the sectors) can still produce robust macro growth.

Echoing the assumptions of Kiley(1999) and Acemoglu (2002), technological advance is assumed to come in two varieties. In the "unskilled labor sector," technical advance comes about from an expansion in the number of intermediate machines specialized for unskilled labor (that is, an increase in M_l). Similarly, in the "skilled labor sector," technical advance means an expansion in the number of intermediate machines specialized for skilled labor (an increase in M_h).

Final goods output produced by different firms is identical, and can be used for consumption, for the production of different intermediate machines, and for research and development to expand the varieties of skill-augmenting and unskilled-augmenting machines. For each time period (suppressing time subscripts) these firms endeavor to maximize:

$$\max_{\{L,H,x_l(j),x_h(k)\}} Y - w_l L - w_h H - \int_0^{M_l} p(j) x_l(j) dj - \int_0^{M_h} p(k) x_h(k) dk \tag{8}$$

where $p(j)$ is the price of machine $x_l(j)$ and $p(k)$ is the price of machine $x_h(k)$.

Endogenous growth theory suggests that research is generally profit motivated. However, modeling purposive research and development effort becomes difficult when prices and factors change over time, as they certainly do when growth is unbalanced. Endogenous growth theory typically assumes that the gains from innovation will flow to the innovator throughout her lifetime, and this flow will depend on the price of the product being produced and the factors required for production at each moment in time.[12] If prices and factors are constantly changing (as they may in an economy where factors evolve endogenously), a calculation of the expected discounted profits from an invention may be impossibly complicated.

[12] For example, the seminal Romer (1990) model describes the discounted present value of a new invention as a positive function of $L - L_R$, where L is the total workforce and L_R are the number of researchers. Calculating this value function is fairly straight-forward if labor supplies of production workers and researchers are constant. If they are not, however, calculating the true benefits to the inventor may be difficult.

To avoid this complication but still gain from the insights of endogenous growth theory, we assume that the gains from innovation last one time period only. More specifically, we assume that intermediate machines are produced either in monopolistic or competitive environments. An inventor of a new machine at time t enjoys monopoly profits for machine production only at t. After this patent rights expire, and subsequent production of this brand of machine is performed by many competitive manufacturers. Whether a machine is produced monopolistically or competitively will be conveyed in its rental price, denoted either as $p(j)$ for a unskilled-labor using machine j or $p(k)$ for a skilled-labor using machine k, and explained in the next sub-section. Also for simplicity, we assume that all machines depreciate completely after use, and that the marginal cost of production is simply unity in terms of the final good.

Assuming for the moment that both technology levels M_l and M_h and labor types L and H are given, an equilibrium can then be characterized as machine demands for $x_l(j)$'s and $x_h(k)$'s that maximize final-good producers' profits (from equation 8), machine prices $p(j)$ and $p(k)$ that maximize machine producers' profits, and factor prices w_l and w_h that clear the labor market.

The first-order conditions for final-good producers yield intermediate-machine demands:

$$x_l(j) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(j)}{\alpha}\right)^{\frac{1}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_h(k) = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} \left(\frac{p(k)}{\alpha}\right)^{\frac{1}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{9}$$

Note that here a greater level of employment of a factor raises the demand for intermediate goods augmenting that factor so long as $\sigma > \alpha$, an idea consistent with Acemoglu's so-called "market-size" effect. We will assume throughout the analysis that this condition is met.

The other first-order conditions for final-good producers illustrate that workers receive their marginal products:

$$w_l = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{10}$$

$$w_h = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{11}$$

3.2 Research

In this section we endogenize the growth paths of M_l and M_h. Researchers expend resources to develop new types of machines, and these resource costs can change over time. We make this modeling choice to stress that unbalanced growth can occur when these costs in different sectors evolve at different rates. We will assume that these costs will depend both on the number of machine types already extant (indexed by M_l and M_h), and on some factor-specific policy variable (denoted by z_l and z_h, and discussed below). Specifically, the up-front cost of developing the blueprint of a new machine, c, is given simply by

$$c\left(\frac{M_l}{z_l}\right) = \frac{M_l}{z_l}$$

for an unskilled labor augmenting machine, and

$$c\left(\frac{M_h}{z_h}\right) = \frac{M_h}{z_h}$$

for a skilled labor augmenting machine. These functional forms illustrate that the costs of invention are negligible when there is little machine variety. As factor-specific technologies grow, however, costs can become increasingly prohibitive.[13].

Given these costs of technological advance, innovating firms must receive some profits from the development of a new technology in order to make research and development worth the expense. As mentioned above, we assume that developers of new machines receive monopoly rights to the production and sale of their machines for only one period. As a result, we must make a distinction between *old* machines (those invented before t) and *new* machines (those invented at t).

Assuming unitary marginal costs of machine production, the revenue generated from new machines of both types are given by the 'value' functions:

$$V_l = (p(j) - 1)\, x_l(j)$$

$$V_h = (p(k) - 1)\, x_h(k)$$

Because demand is isoelastic, the price which maximizes monopolists' profits equals $1/\alpha$ for both skill- and unskilled-augmenting machines, so that demand for *new* intermediate machines (those invented at t) are notated simply as:

$$x_{l,new}(j) = x_{l,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,new}(j) = x_{h,new} = \alpha^{\frac{2}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{12}$$

On the other hand, because older machines are competitively produced, their prices equal unitary marginal costs, so that demand for *old* intermediate machines (those invented before t) are:

[13]This approach of varying the cost of research echoes the leader-follower model illustrated in Barro and Xala-i-Martin 2003), where costs depend on the distance from the frontier of general knowledge

$$x_{l,old}(j) = x_{l,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}}$$

$$x_{h,old}(j) = x_{h,old} = \alpha^{\frac{1}{1-\alpha}} \left[(A_l L)^{\sigma} + (A_h H)^{\sigma}\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \tag{13}$$

Thus factor-specific TFPs given by equation (6) can be re-written as an aggregation of two kinds of machines, illustrating the cumulation of all past and current innovation. If $M_{z,old}$, $M_{z,new}$, and M_z are, respectively, the number of existing old, new and total machine-types used by factor z, we can write factor production as:

$$A_l \equiv \int_0^{M_l} \left(\frac{x_l(j)}{L}\right)^{\alpha} dj = \left[\int_0^{M_{l,old}} x_{l,old}\,(j)^{\alpha}\, dj + \int_{M_{l,old}}^{M_l} x_l\,(j)^{\alpha}\, dj\right] (1/L)^{\alpha} =$$
$$\frac{M_{l,old}\; x_{l,old}^{\alpha} + M_{l,new}\; x_{l,new}^{\alpha}}{L^{\alpha}} \tag{14}$$

$$A_h \equiv \int_0^{M_h} \left(\frac{x_h(k)}{H}\right)^{\alpha} dk = \left[\int_0^{M_{h,old}} x_{h,old}\,(k)^{\alpha}\, dk + \int_{M_{h,old}}^{M_h} x_{h,new}\,(k)^{\alpha}\, dk\right] (1/H)^{\alpha} =$$
$$\frac{M_{h,old}\; x_{h,old}^{\alpha} + M_{h,new}\; x_{h,new}^{\alpha}}{H^{\alpha}} \tag{15}$$

Substituting the monopoly price into our value functions yield:

$$V_l = \left[\frac{1-\alpha}{\alpha}\right] x_{l,new}$$

$$V_h = \left[\frac{1-\alpha}{\alpha}\right] x_{h,new}$$

where $x_{l,new}$ and $x_{h,new}$ are given by (12). Finally, an individual is free to research, guaranteeing that:

$$V_l\,(L, H, A_l, A_h) \leq c\left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{16}$$

$$V_h\,(L, H, A_l, A_h) \leq c\left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{17}$$

If resource costs of research were actually *less* than discounted profits, entry into research would occur, driving technology levels, and hence costs, up. I assume this happens quickly, so that valuations never exceed costs in any time period. Further, since applied research is irreversible (a society cannot forget how to make something once it is learned), the variety of machines remains unchanged when the inequalities in (16) or (17) do not bind with equality.

The levels of our policy variables z_l and z_h in the economy are key determinants of the costs of developing new "production processes;" higher levels of z_u lower the costs of developing intermediate machines which complement factor u. Conceivably these variables are functions of many possible factors, such as government policies, or technological diffusion from other countries. For the moment let us assume that these variables simply grow at an exogenously steady rate:

$$g = \frac{z_{l,t+1} - z_{l,t}}{z_{l,t}} = \frac{z_{h,t+1} - z_{h,t}}{z_{h,t}}$$

where $g > 0$ is the growth factor. Thus we see that (16) and (17) can also illustrate barriers to technology adoption - if z_l and/or z_h are too small, factor-specific technological growth cannot happen.

The steady-state can be characterized as one where the share of labor devoted to each sector (skilled and unskilled) remains fixed, while output, the policy variables z_l and z_h, the varieties of skilled and unskilled machines, and wages all grow at the same rate, g. This will occur so long as equations (16) and (17) hold with strict equality. But as these inequalities imply there may be a considerable period of time when growth is *unbalanced*; this would occur if only one of the equations held with equality. What kind of unbalanced growth is likely to unfold will depend on a number of things, including the available supply of different factors (a relatively large L for example raises V_l and thus increases the chance that growth will be unskill-biased) and the relative "skewness" of the policy variables (a relatively large z_l for example lowers c_l and likewise increases the chance for unskill-biased growth).

No doubt unbalanced growth will be slower than balanced steady-state growth, but it seems logical that growth in the *bigger* sector will produce faster growth than growth in the smaller sector.[14] This indeed is the essence of the appropriate technology story - typically it involves a story of factor abundance. By its logic, a country awash with throngs of unskilled labor would do well to develop and adopt technologies readily employable by them. The tragedy stressed in this tale often involves the nature of the technology frontier - because cutting-edge technologies produced by wealthy nations tend to be skill-intensive, developing nations often inherit a lot of skill-intensive technologies (Acemoglu and Zilibotti 2001). In our simple model this may be reflected by a large (z_h/z_l); the consequence of this is that poor countries end up developing technologies for which they are structurally ill-suited, resulting in anemic macro growth.[15]

[14] If $\frac{\Delta a}{a} = g$ and $\frac{\Delta b}{b} = 0$, $\frac{\Delta(a+b)}{a+b} = \frac{\Delta a}{a+b}$, which is smaller than, but converges to, g. The smaller is b relative to a, the closer will this growth be to g.

[15] The development literature is filled with anecdotal evidence of this technology-skill mismatch, highlighted in Todaro and Smith's seminal text. "Gleaming new factories with the most modern and sophisticated machinery and equipment are a common feature of urban industries while idle workers congregate outside the factory gates." (pp. 256 in Todaro and Smith 2006).

At the same time, there is recognition among development economists of the importance of skill accumulation in economic growth. The centrality of human capital in economic development is so established that most economists now treat education and modernity as going hand in hand.[16] From this perspective, a country's relative abundance in unskilled labor scarcely matters; the skill-intensive path is the *only* viable path to sustainable progress.

This paper suggests that forces that change the factors of production themselves are an important part of our answer to the question of which is the more appropriate growth path. Specifically, changes in the relative rewards to factors due to technological developments surely will alter the incentives to accumulate education or to remain an unskilled laborer. From the model we can write the "skill premium," the skilled wage relative to the unskilled wage, as

$$\frac{w_h}{w_l} = \left(\frac{\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}}{\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}} \right)^{\frac{\sigma-\sigma\alpha}{1-\sigma\alpha}} \cdot \left(\frac{H}{L} \right)^{\frac{\sigma-1}{1-\sigma\alpha}} \tag{18}$$

In the absence of any demographic response, skill-bias technological growth will raise the skill premium (by raising $M_{h,new}$), while unskill-bias technological growth will lower it (by raising $M_{l,new}$). But surely if unskill-intensive growth lowers the relative returns to skill, this will induce people to remain unskilled and so not accumulate human capital. Conversely, increases in the returns to skill should induce individuals to increase human capital, and thus lower fertility rates through the quality-quantity tradeoff. Indeed, from the last section we suggest that the more responsive these factors are to changes in their relative returns, the more likely will skill-biased technological growth yield greater income per capita growth. The question we now want to ask is if this endogenous growth model can be combined with a fairly simple model of demography that can test these possibilities. The next sections endeavor to do precisely that.

3.3 Endogenous Demography

To capture the symbiotic relationship between technologies and factors, we introduce households into the model in an over-lapping generations framework, where individuals have two stages of life: young and old. Only old people are allowed to make any decisions regarding demography. Specifically, the representative household is run by an adult who maximizes her income by deciding two things: how many children to have (denoted by n) and the fraction of these children who will receive an education (denoted by e).

An individual born at time t works either as an unskilled laborer (earning the unskilled wage w_l), or as a skilled laborer (using their human capital to earn the skilled wage w_h). Either way, all earnings are relinquished to the parent. The individual becomes old at $t + 1$. At this point

[16] By one recent paper's account, "Anything that harms the accumulation of human capital harms our economic well-being" (Remler and Pema 2009). For a brief history of the study of human capital see Ehrlich and Murphy 2007.

she decides how many children to have herself, and the fraction of these children that will get an education and work as skilled workers. After incurring the costs of child rearing, she consumes all the income generated by her children. After this she expires and exits the economy.

Note that here offspring are simply tools of income generation.[17] That is, we can say that the income-maximizing adult chooses a pair of (n,e) to solve

$$\max_{n,e}\ \tilde{w} + w_l(1-e)n + w_h h(n,e) - \tilde{w}c(n,e)$$

where $h(\cdot)$ is the function denoting human capital, $c(\cdot)$ is the function denoting child-rearing costs, and $\tilde{w}$ is the wage of the parent (who could be either a skilled worker or an unskilled worker, depending on what *her* parent chose for her last time period). The first three terms are respectively her income, her unskilled children's income, and her skilled children's income. The last term is her forgone income in raising children.

To merge this simple objective function for households into our model in a tractable way, we require that $h(\cdot)$ exhibits diminishing returns in its inputs, and that $c(\cdot)$ is convex. Specifically, we will assume the following simple forms:

$$\max_{n,e}\ \tilde{w} + w_l(1-e)n + w_h\,(ne)^{1/2} - \tilde{w}e^2 - \tilde{w}\,(1+ne)^2 \tag{19}$$

$$\tilde{w} = w_l^{1/2} w_h^{1/2} \tag{20}$$

Note that $\tilde{w}$ is simply an average of skilled and unskilled wages. This is simply to reflect that the opportunity cost of child rearing for adults will depend on both wage rates, without having to keep track of who precisely is a skilled adult and who is not.[18]

The first-order condition for the number of children is:

$$w_l(1-e) + \frac{1}{2}w_h\left(\frac{e}{n}\right)^{1/2} = 2\tilde{w}\left(e + n + ne^2\right) \tag{21}$$

The left-hand side illustrates the marginal benefit of an additional child, while the right-hand side denotes the marginal cost. At the optimum, the gains in income from an extra unskilled worker in the family *and* the additional source of skilled income precisely offsets the foregone income that results from child-rearing.

The first order condition for education is:

$$\frac{1}{2}w_h\left(\frac{1}{ne}\right)^{1/2} = w_l + 2\tilde{w}\,(1+ne) \tag{22}$$

[17] This echoes Moav (2005), who models parents that decide both the number of children and the level of human capital of each child in order to simply maximize their potential income. Of course households may wish to invest in their children for reasons other than to simply maximize income. This more general possibility is explored in the Appendix.

[18] When we do keep track of both types of workers, as in Kremer and Chen (2002), the results do not change.

Again, the left-hand side is the marginal benefit and the right-hand side the marginal cost. At the optimum, the gains received from the added skilled income offsets the foregone unskilled- and adult-income requisite for giving more children an education.

Note that this simple optimization problem does capture the quality-quantity tradeoff inherent in child production. For example, rising unskilled wages induces households to increase their fertility; the rise in child-rearing costs this produces however will also incentivise households to lower their education endowment (see however the appendix, where this quality-quantity tradeoff can be produced using a more general objective function).

Finally, completing the model requires us to relate fertility and education rates to aggregate levels of unskilled labor and human capital. Here we propose:

$$L = Nn(1 - e) \tag{23}$$

$$H = (Nne)^{1/2} \tag{24}$$

where N is simply the adult population.

Combining this model of demography with our model of biased technologies is straightforward. Through the simultaneous solving of (10), (11), (14), (15), (16), (17), (21), (22), (23), and (24), a unique set of variables w_l, w_h, A_l, A_h, M_l, M_h, n, e, L, and H can be determined for every time period.[19] We can perhaps synopsize our findings by initially focusing only on the economy's choice of e and M_h. If an adult expects researchers to develop new skill-biased technologies (and so to increase w_h), she will want to endow her children with more human capital. Similarly, if researchers anticipate a larger pool of human capital, they may wish to invent and build new skill-intensive machines, raising $M_{h,new}$ and thus M_h overall. Consequently we can plot the two "reaction functions" of each group as two upward-sloping curves; the development of new skill-using machines and the accumulation of skills are strategic complements. From the intersection of these reaction curves we find the unique simultaneous solution of the level of education and the new skill-biased technical coefficient. This is done in Figure 5. We can similarly plot two upward-sloping curves to determine an economy's choice of n and M_l.

To summarize, potential researchers look to the skill composition of the workforce (something influenced by households) to determine the direction and scope of technical change. Households look to wages (something influenced by researchers) to determine the levels of skilled and unskilled workers. Together they jointly determine the overall composition of the economy.

[19]This 10-by-10 system is reiterated with more detail in the Appendix.

4 "Appropriate" Growth Paths for a Developing Country - Some Simulations[20]

Now that we have a model that endogenizes the growth paths of both technologies and factors, we may better assess the appropriateness of alternative development paths. Let us consider a hypothetical developing country endowed with a fairly sizeable amount of unskilled labor and a modest amount of skilled labor. We can then test the effects of unbalanced growth by allowing only either unskilled-labor technology or skilled-labor technology to rise, run the "horse-race," and compare the two paths. Each simulation is run for ten time periods.

4.1 Case 1 - Simulation with Exogenous Unbalanced Growth[21]

Our first horse-race is where we simply have either A_l or A_h grow exogenously, and compare the two paths. In other words, we ignore our discussion about endogenous technical growth in section 3.2 for the moment, and assume that unbalanced growth happens simply as some exogenous process, such as through technological diffusion from other countries (see Appendix A for the system of equations being solved each time period). Specifically, each technological parameter grows 2% each period.

Figure 6 illustrates the results of this simulation. Red dotted lines are where only A_l grows; Blue solid lines are where only A_h grows. Unskilled-biased growth puts downward pressure on skill premia, and this induces a rise in fertility and a fall in education. Skill-biased growth on the other hand raises skill premia, inducing a fall in fertility and a rise in education. These demographic shifts induce a substantial rise in unskilled labor in the former case and a substantial fall in the latter. Skilled labor on the other hand does not move very much at all - with unskilled-bias growth, an initial slight rise in H is followed by a fall in H! This is simply because the quantity-quality population variables move in opposing directions, and this keeps skilled labor fairly constant since it depends on both (see eq 24).

However, it is clear that this case creates big changes in *relative* factors. That is, H/L falls with unskilled-bias growth, and rises with skilled-bias growth. Recall from our discussion in section 2.2 that the latter means there is relative growth in the more productive factor (H is more productive even though we start with $A_l = A_h$ because it is scarcer than L), and this should be a boost to overall income. On top of this, the overall population grows with unskilled-bias growth and falls with skilled-bias growth.

[20]Note that for the lessons of the simulations to hold, we require only that $0 < \alpha < \sigma < 1$. This simply means that factors of production must be substitutable "enough." Specifically we assume that $\alpha = 0.33$ and $\sigma = 0.5$.

[21]For this case initial values of A_l and A_h are set to 0.5 and N is set to 5. With n normalized to one to represent a stable population, $e = 0.188$, and this gives us initial factor endowments of $L = 4.06$ and $H = 0.97$, and initial wages of $w_l = 0.74$ and $w_h = 1.54$.

For these two reasons, GDP per person, given by $y = Y/N$, grows faster with skill-biased growth than with unskill-biased growth. Thus, even though unskilled technologies augment a much larger segment of the economy, the dynamic effects of growth of these technologies undermine overall performance. Growth in unskilled labor generates growth in the entire population, with no beneficial feedback through market-size effects for researchers (since technologies grow exogenously). Thus the lessons of section 2.2 seem to apply here - if technologies can be considered to grow exogenously and factors are responsive to technological changes, a skill-intensive technological path can produce more income per person than the alternative path.

4.2 Case 2 - Simulation with Endogenous Unbalanced Growth Starting with $M_l \approx M_h$[22]

In this case we use the full system of equations that jointly solve for technological levels and for demographic variables. That is, both technologies and factors are in this case endogenized (see Appendix A for the full system of equations). Specifically, for unskilled-intensive growth, we set z_l such that $V_l = c_l$ at the start of the simulation. Then we simply have z_l grow 2% each period, inducing research that produces new varieties of unskilled labor-using machines. For skill-intensive growth we do the same to z_h, V_h and c_h.

Figure 7 illustrates the results of this simulation. Immediately we can see a big divergence between the two growth paths. Unskilled-intensive growth generates big fertility increases, which in turn produce large increases in unskilled labor. Interestingly, unskilled-intensive growth significantly lowers education rates, but skilled labor rises as well. This is simply because here dramatic increases in fertility more than offset declines in education - as a result *both* labor types end up rising.

What is striking here is that unskilled-intensive growth produces very robust growth in GDP per person, while skill-intensive growth keeps GDP per person nearly constant. This may be surprising at first, because like Case 1, unskilled-intensive growth lowers H/L and we have said that H tends to be the more productive factor. And it bucks conventional wisdom, which suggests that decreaes in education can be destructive for the overall macro economy. But the fall in relative skilled labor comes from increases in L (as opposed to decreases in H); this is good for future growth due to the increasing market size effects that researchers observe.

Another surprising result here is that A_h grows robustly only with unskilled-intensive growth, even though in this case M_h remains constant. Ironically, skill-intensive growth keeps A_h almost totally stagnant! Again, this has everything to do with beneficial market-size effects. As skilled labor rises with increases in fertility, producers are keen on making these workers more productive.

[22]For this case initial levels of machines are set to $M_l = 0.3$ and $M_h = 0.35$. This gives us initial values of A_l and A_h of 0.5, the same balanced situation as in Case 1. All other initial values are hence the same as Case 1.

So even though the number of skill-using machines remain fixed with unskilled-intensive growth, the quantity demanded for *existing* machines substantially rises with the increase in H.

So even though unskilled-bias growth creates a population boom, it is accompanied by an income boom as well. The rise in labor makes overall productivity rise both through increases in A_l (by making V_l rise and thus inducing researchers to invent new unskilled labor using machines) and increases in A_h (by raising the demand for existing skilled labor using machines). Skilled-bias growth on the other hand lowers fertility and increases education; this keeps both labor types low and so market-size effects can not thrive. Technologies as a result remain virtually stagnant. This case suggests that the high fertility and child-labor participation rates we observe for poorer nations may actually contribute to future economic prosperity, not hinder it as the prior case suggests.

4.3 Case 3 - Simulation with Endogenous Unbalanced Growth Starting with $M_l < M_h$

While the above case implies that unskilled-intensive growth produces more per capita income in the long-run than the alternative, we should acknowledge that this case is based on the extreme assumption that $A_h = A_l$ at the start. But we could very well have the case where $A_h > A_l$. This is especially true if we have a fairly broad definition of H. For example, when H is considered those with primary education or more, skilled-labor productivity tends to be larger for unskilled-labor productivity for most countries (see Table 1). As mentioned in section 2.1, wealthier nations also tend to have higher relative productivity levels for skilled workers.

Our final case explores this possibility by endowing skilled-labor with many more machine varieties than unskilled labor, thus making skilled labor inherently more productive.[23] Figure 8 illustrates this case.

Note that now there is fairly sizable growth in H with skill-intensive growth. This produces a lot more output because M_h already starts out pretty high, and it contributes to even more growth in M_h through beneficial market-size effects. Ultimately though, growth in H peters out. Why does this happen? The fall in fertility, coupled with the stagnation of education, forces this outcome.

Interestingly, even though skilled labor is much more productive than unskilled labor, the unskilled technological path will eventually win the horse race if we let it run long enough. With skill-intensive growth, the quantity-quality tradeoff and the forces of demographic momentum sow the seeds for this path's eventual slowdown. The fall in fertility that inevitably results (as kids get more and more expensive) anchors the growth in H so the market size effects with this path inevitably peter away.

[23]Specifically, we set initial levels of M_l and M_h to 0.3 and 2, respectively. Again, after normalizing n to 1, we get initial values of $e = 0.36$, $L = 3.22$, $H = 1.34$, $w_l = 1.56$, and $w_h = 4.85$.

5 Conclusion

This paper has suggested that modeling the simultaneity of factors and technologies can help us better understand the process of macroeconomic growth. Unlike approaches that credit either technological progress (Christensen and Cummings 1981) or factor accumulation (Young 1995) alone for economic success, the interaction of both can lend us new insights over what development path will breed the greatest rewards.

We see that the answer depends on the structure of the macroeconomy. Generally, a skill-intensive path will generate more benefits the more productive skilled labor is. It also produces more benefits the more *responsive* are factors to technological changes, provided there is no or limited feedback from these changes on technologies. This is because the falling population growth caused by skill-intensive growth, normally a boom to income per capita, would hurt economic growth if technologies are locally endogenous. Thus skill-biased technological diffusion, of the kind generated by the world-wide pervasiveness of skill-intensive technologies (Berman and Machin 2000; Berman, Bound and Machin 1998) can generate robust growth *because* of its exogenous nature; and this despite its apparent inappropriateness due to a low endowment of H.

The bottom line is that the proper path to macro prosperity depends on lots of things - here we provide only the broadest brush-strokes delineating the major concerns. Much depends on the country-specific context. For example, a country with limited access to public education but with a thriving inventive and entrepreneurial spirit may well be advised not to simply copy the ways of the "West," but rather to take the unskilled labor route. From our discussions above, such an economy would not be able to raise H much to take advantage of skill-intensive growth; on the other hand, the potential inventors in the economy would be ready to make the growing workforce caused by unskilled-intensive growth even more productive.

Bibliography

Acemoglu, Daron (1998), "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *The Quarterly Journal of Economics*, Vol. 113, pp. 1055-1089.

Acemoglu, Daron (2002), "Directed Technical Change," *Review of Economic Studies*, Vol. 69, pp. 781-809.

Acemoglu, Daron (2008), *Introduction to Modern Economic Growth*. Princeton, N.J.: Princeton University Press.

Acemoglu, Daron and Fabrizio Zilibotti (2001), "Productivity Differences," *Quarterly Journal of Economics*, Vol. 116, pp. 563-606.

Autor, David, Lawrence Katz and Alan Krueger (1998), "Computing Inequality: Have Computers Changed the Labor Market?" *Quarterly Journal of Economics*, Vol. 113, pp. 1169-1213.

Basu, Susanto and David N. Weil (1998), "Appropriate Technology and Growth," *The Quarterly Journal of Economics*, Vol. 113, pp. 1025-1054.

Barro, Robert J. and Xavier Sala-i-Martin (2003), *Economic Growth*, 2nd Edition. Cambridge: MIT Press.

Becker, Gary S. and Robert J. Barro (1988), "A Reformulation of the Economic Theory of Fertility," *The Quarterly Journal of Economics*, Vol. 103, pp. 1-25.

Becker, Gary S. and H. Gregg Lewis (1973), "On the Interaction between the Quantity and Quality of Children," *The Journal of Political Economy*, Vol. 81, S279-S288.

Berman, Eli, John Bound and Stephen Machin (1998), "Implications of Skill-Biased Technological Change: International Evidence," *The Quarterly Journal of Economics*, Vol. 113, pp. 1245-1279.

Berman, Eli and Stephen Machin (2000), " Skill-Biased Technology Transfer: Evidence of Factor-Biased Technological Change in Developing and Developed Countries," unpublished manuscript.

Bils, Mark and Peter J. Klenow (2000), "Does Schooling Cause Growth," *The American Economic Review*, Vol. 90, pp. 1160-1183.

Caselli, Francesco and Wilbur J. Coleman II (2006), "The World Technology Frontier," *The American Economic Review*, Vol. 96, pp. 499-522.

Chari, V. V. and Hugo Hopenhayn (1991), "Vintage Human Capital, Growth, and the Diffusion of New Technology," *The Journal of Political Economy*, Vol. 99, pp. 1142-1165.

Ciccone, Antonio and Giovanni Peri (2005), "Long-Run Substitutability between More and Less Educated Workers: Evidence from U.S. States 1950-1990." *Review of Economics and Statistics*, Vol. 87, pp. 652-63.

Christensen, Laurits R. and Diane Cummings (1981), "Real Product, Real Factor Input, and Productivity in the Republic of Korea, 1960-1973," *The Journal of Development Economics*, Vol. 8, pp. 285-302.

Edwards, Eric and Nina Pavcnik (2005), "Child Labor in the Global Economy," *Journal of Economic Perspectives*, Vol. 19, pp. 199-220.

Ehrlich, Issac and Kevin M. Murphy (2007), "Why Does Human Capital Need a Journal?" *Journal of Human Capital*, Vol. 1, pp. 1–5.

Galor, Oded and Omer Moav (2000), "Ability-Biased Technological Transition, Wage Inequality and Growth," *The Quarterly Journal of Economics*, Vol. 115, pp. 469-498.

Galor, Oded and Andrew Mountford (2006), "Trade and the Great Divergence: The Family Connection," *American Economic Review Papers and Proceedings*, Vol. 96, pp. 299-303.

Galor, Oded and Andrew Mountford (2008), "Trade and the Great Divergence: Theory and Evidence,"*Review of Economic Studies*, Vol. 75, pp. 1143-1179.

Galor, Oded and David N. Weil (2000), "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *The American Economic Review*, Vol. 90, pp. 806-828.

Grossman, Gene M. and Elhanan Helpmann (1991), *Innovation and Growth in the Global Economy*. Cambridge, MIT Press.

International Labor Organization (2002), *Every Child Counts: New Global Estimates on Child Labor*. Geneva: ILO.

Jones, Charles I. (2002), "Sources of U.S. Economic Growth in a World of Ideas." *American Economic Review*, Vol. 92, pp. 220-239.

Katz, Lawrence F. and Kevin M. Murphy (1992), "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *The Quarterly Journal of Economics*, Vol. 107, pp. 35-78.

Kiley, Michael T. (1999), "The Supply of Skilled Labor and Skill-Biased Technological Progress." *Economic Journal*, Vol. 109, pp. 708-724.

Kremer, Michael and Daniel L. Chen (2002), "Income Distribution Dynamics with Endogenous

Fertility." *Journal of Economic Growth*, Vol. 7, pp. 227-258.

Moav, Omer (2005), "Cheap Children and the Persistence of Poverty," *Economic Journal*, Vol. 115, pp. 88-110.

Mokyr, Joel (1999), *The British Industrial Revolution - An Economic Perspective*, 2nd edition. Westview Press, Boulder, CO.

Redding, Stephen (1996), "The Low-Skill, Low-Quality Trap: Strategic Complementarities between Human Capital and Research and Development," *The Economic Journal*, Vol. 106, pp. 458-470.

Remler, Dahlia K. and Elda Pema (2009), "Why Do Institutions of Higher Education Reward Research While Selling Education?" NBER Working Paper No. 14974.

Romer, Paul M. (1990), "Endogenous Technological Change," *The Journal of Political Economy*, Vol. 98, The Problem of Development: A Conference of the Institute for the Study of Free Enterprise Systems, pp. S71-S102.

Stokey, Nancy L. (1988), "Learning by Doing and the Introduction of New Goods," *The Journal of Political Economy*, Vol. 96, pp. 701-717.

Todaro, Michael P. and Stephen C. Smith (2006), *Economic Development*, Ninth Edition, Pearson Addison Wesley.

Xu, Bin (2001), "Endogenous Technology Bias, International Trade, and Relative Wages," University of Florida Working Paper.

Young, Alwyn (1993), "Substitution and Complementarity in Endogenous Innovation," *The Quarterly Journal of Economics*, Vol. 108, pp. 775-807.

Young, Alwyn (1995), "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience," *The Quarterly Journal of Economics*, Vol. 110, pp. 641-680.

Appendix A: Simulations

If technologies are exogenously determined, the simulation pre-determines A_l and A_h, and solves the following system of equations for w_l, w_h, n, e, L, and H for each time period.

$$w_l = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{25}$$

$$w_h = [(A_l L)^\sigma + (A_h H)^\sigma]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{26}$$

$$w_l(1-e) + \frac{1}{2} w_h \left(\frac{e}{n}\right)^{1/2} = 2\tilde{w}\left(e + n + ne^2\right) \tag{27}$$

$$\frac{1}{2} w_h \; 1ne)^{1/2} = w_l + 2\tilde{w}\,(1 + ne) \tag{28}$$

$$L = Nn(1-e) \tag{29}$$

$$H = (Nne)^{1/2} \tag{30}$$

If on the other hand technologies are endogenously determined by the process discussed in section 3.2, the following equations are solved for $M_{l,new}$, $M_{h,new}$, A_l, A_h, w_l, w_h, n, e, L, and H for each time period.

$$\left(\frac{1}{1+r}\right)\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}\left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_l^{\frac{1-\sigma}{\alpha-1}} L^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{l,old} + M_{l,new}}{z_l}\right) \tag{31}$$

$$\left(\frac{1}{1+r}\right)\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}\left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{(1-\alpha)\sigma}} A_h^{\frac{1-\sigma}{\alpha-1}} H^{\frac{\sigma-\alpha}{1-\alpha}} \leq \left(\frac{M_{h,old} + M_{h,new}}{z_h}\right) \tag{32}$$

$$A_l = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{l,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{l,new}\right]\left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_l^{\frac{(\sigma-1)\alpha}{1-\alpha}} L^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{33}$$

$$A_h = \left[\alpha^{\frac{\alpha}{1-\alpha}} M_{h,old} + \alpha^{\frac{2\alpha}{1-\alpha}} M_{h,new}\right]\left((A_l L)^\sigma + (A_h H)^\sigma\right)^{\frac{(1-\sigma)\alpha}{(1-\alpha)\sigma}} A_h^{\frac{(\sigma-1)\alpha}{1-\alpha}} H^{\frac{\alpha(\sigma-1)}{1-\alpha}} \tag{34}$$

$$w_l = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_l^\sigma L^{\sigma-1} \tag{35}$$

$$w_h = \left[(A_l L)^\sigma + (A_h H)^\sigma\right]^{\frac{1-\sigma}{\sigma}} A_h^\sigma H^{\sigma-1} \tag{36}$$

$$w_l(1-e) + \frac{1}{2} w_h \left(\frac{e}{n}\right)^{1/2} = 2\tilde{w}\left(e + n + ne^2\right) \tag{37}$$

$$\frac{1}{2} w_h \left(1ne\right)^{1/2} = w_l + 2\tilde{w}\left(1 + ne\right) \tag{38}$$

$$L = Nn(1-e) \tag{39}$$

$$H = (Nne)^{1/2} \tag{40}$$

(31) and (32) illustrate the benefits and costs of innovation; (33) and (34) are factor-specific TFP levels as functions of the demand for old and new machines and factors of production; (35) and (36) are wages; (37) and (38) are the benefits and costs of having children and educating them; (39) and (40) describe how fertility and education choices translate into aggregate factors of production. Note that if either of the first two equations holds with strict inequality, the algorithm sets the value of M_{new} to zero and simply solves the the rest of the system.

Appendix B: Human Capital As Consumption As Well As Investment

Section 3.3 modeled human capital strictly as an investment good. That is, in the model parents cared about the education only insofar as it raised their earnings. However, we must concede that education can bring with it all sorts of benefits other than simply increasing one's earnings potential. These include benefits in health, child-rearing, marriage, and consumption of new goods (Becker and Murphy 2007). Thus human capital can also be considered a *consumption* good, one that tends to rise with income.[24] To incorporate this general case, we can model utility as:

$$\max_{n,e} U = \min\left[A \cdot income\left(n,e\right), B \cdot h\left(n,e\right)\right] \tag{41}$$

where income is given by the maximized expression in (19), $h(n,e)$ is given simply by $(ne)^{1/2}$, as described in section 3.3, and $A > 0$ and $B > 0$ are weighting coefficients. Here income and human capital are considered perfect complements in utility. If income is of sufficient size, any rise in wages (due to technological growth) must be accompanied by human capital increases in order for utility to rise. So, we can consider this utility function as a more general case that nests the specific case described in section 3, where technologies and wages are low enough such that $A \cdot income < B \cdot h$ for all values in the simulation (and where $A = 1$). In that case, households only care about maximizing income.

Once income reaches a certain level, however (where $A \cdot income \approx B \cdot hk$), human capital becomes a consumption good as well as an investment good. At this point, any wage increase will be accompanied by an increase in human capital. Indeed, we can now consider this case a constrained optimization problem - once the threshold income is reached, households continue to maximize income, but now they are constrained by the condition that human capital must rise right along with income in a perfectly balanced way. We can thus set up for the household problem the Lagrangian:

$$Ł = income(n,e) - \lambda\left[A \cdot income\left(n,e\right) - B \cdot (ne)^{1/2}\right] \tag{42}$$

Our three first order conditions are

$$\frac{\partial Ł}{\partial n} = A \cdot \frac{\partial income}{\partial n} - \lambda\left[A \cdot \frac{\partial income}{\partial n} - B \cdot \frac{1}{2}\left(\frac{e}{n}\right)^{1/2}\right] = 0 \tag{43}$$

$$\frac{\partial Ł}{\partial e} = A \cdot \frac{\partial income}{\partial e} - \lambda\left[A \cdot \frac{\partial income}{\partial e} - B \cdot \frac{1}{2}\left(\frac{n}{e}\right)^{1/2}\right] = 0 \tag{44}$$

[24]This is the implicit assumption of demography models where parents educate their children out of pure altruism. Acemoglu (2008) has labeled this a kind of "warm glow" preference.

$$A \cdot income = B \cdot (ne)^{1/2} \tag{45}$$

Now, for each time period we can solve the system of equations given by (10), (11), (14), (15), (16), (17), (23), (24), (43), (44), and (45) for w_l, w_h, A_l, A_h, M_l, M_h, L, H, n, e and λ.

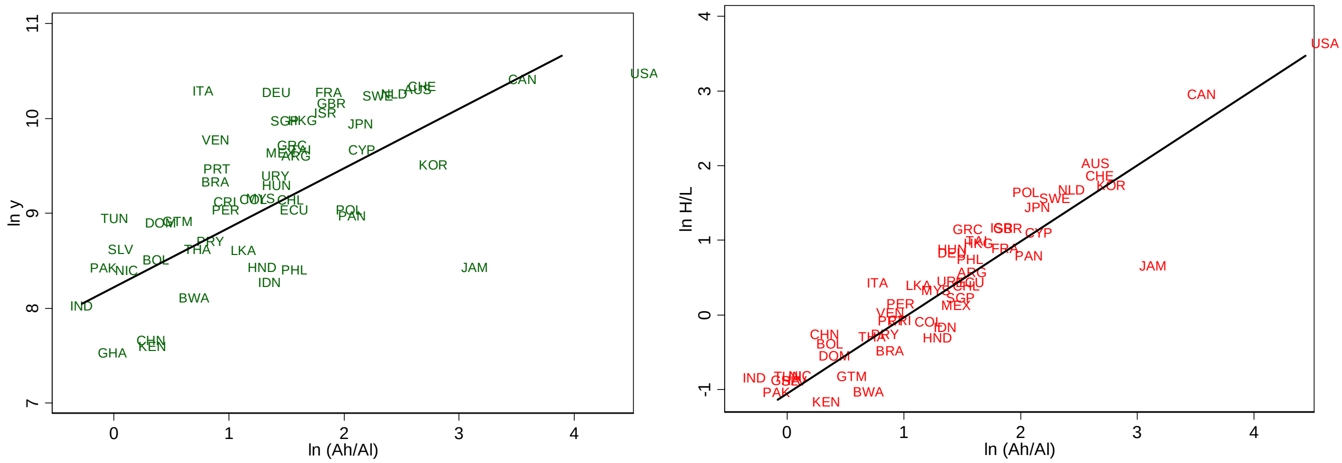
So what does treating human capital as a consumption good in this type of approach illustrate? As it turns out, this more general case does not alter any of our qualitative conclusions from section 4 (simulations not illustrated). If technologies are endogenous, unskilled technological growth still produce dramatic rises in L and H which in turn generate robust factor productivity growth in both sectors, even when $A \cdot income \approx B \cdot hk$. The reason for this is fairly clear - human capital can be increased by raising the fraction of young who receive an education (e), or by raising the number of young (n). With unskilled technological growth, households choose to do the latter. Thus, when w_l rises, households can increase fertility while keeping education fairly constant, and this allow both income and human capital to rise at the same pace.

On the other hand, skill-biased technological growth induces a rise in education and a fall in fertility, through the quantity-quality tradeoff. Again, just like in the original model, this tends to keep the growth in overall human capital fairly limited, and so subsequent skill-biased technologies can not grow very robustly. As a result both income and human capital grow much slower than with unskilled biased growth.

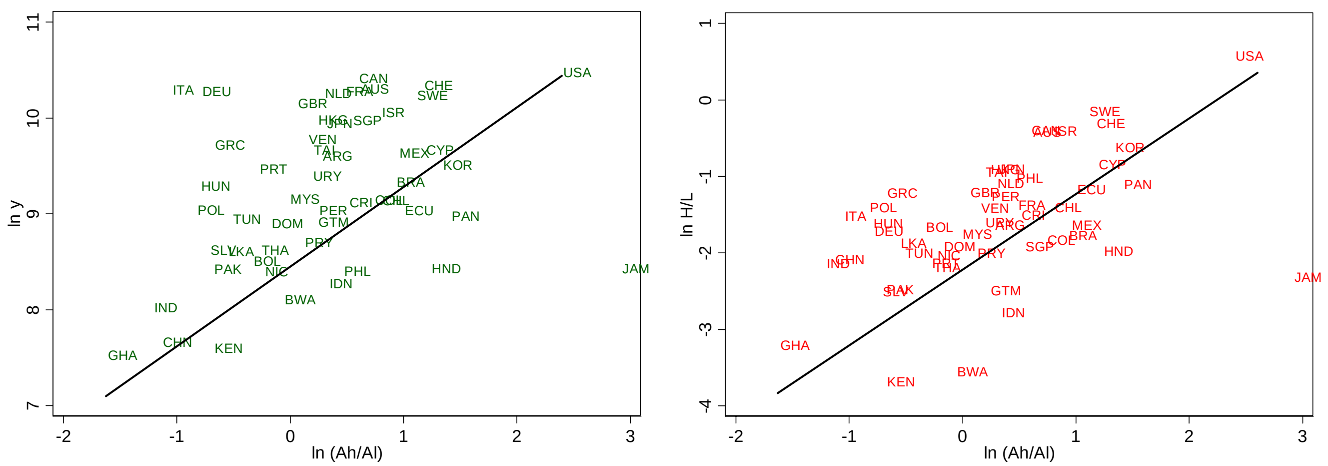
In summary, this general case suggests that the findings of the paper are not very sensitive to the particular assumptions concerning household demographic choices. Many different household objective functions will generate similar conclusions.

Figure 1 – Relative Technologies vs. Relative Factors and Output (σ = 0.5)

H = primary school or more completed



H = secondary school or more completed



H = college or more completed

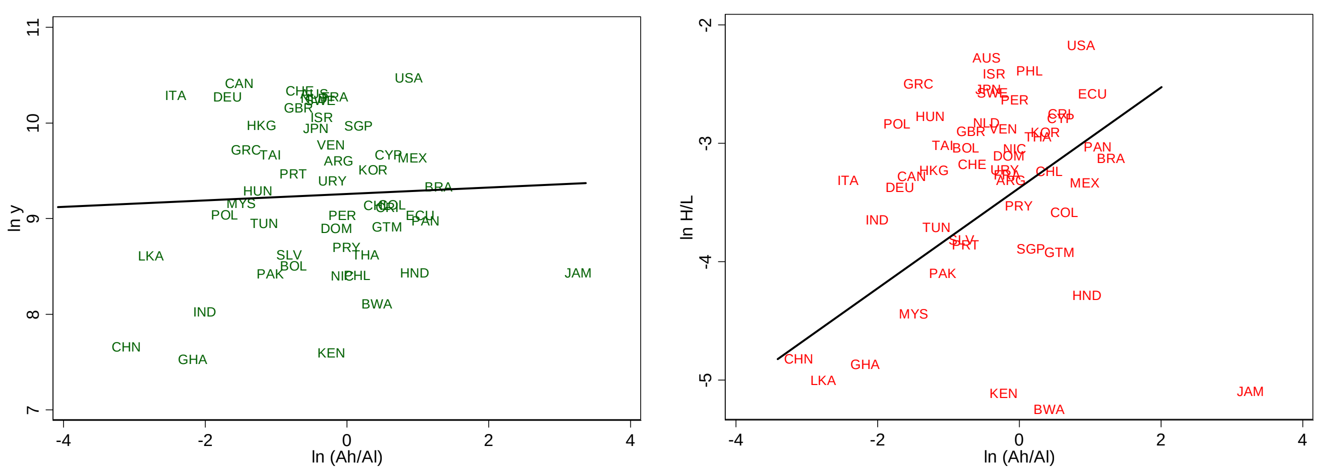
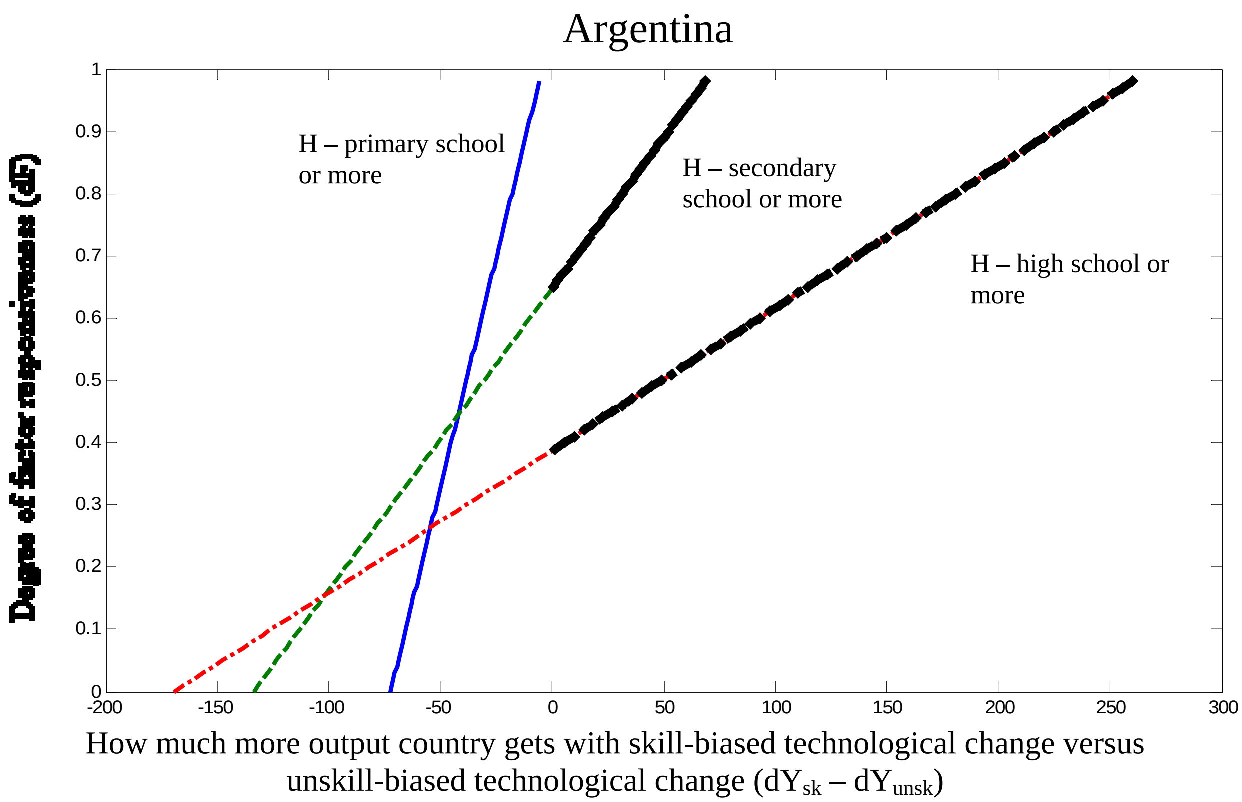
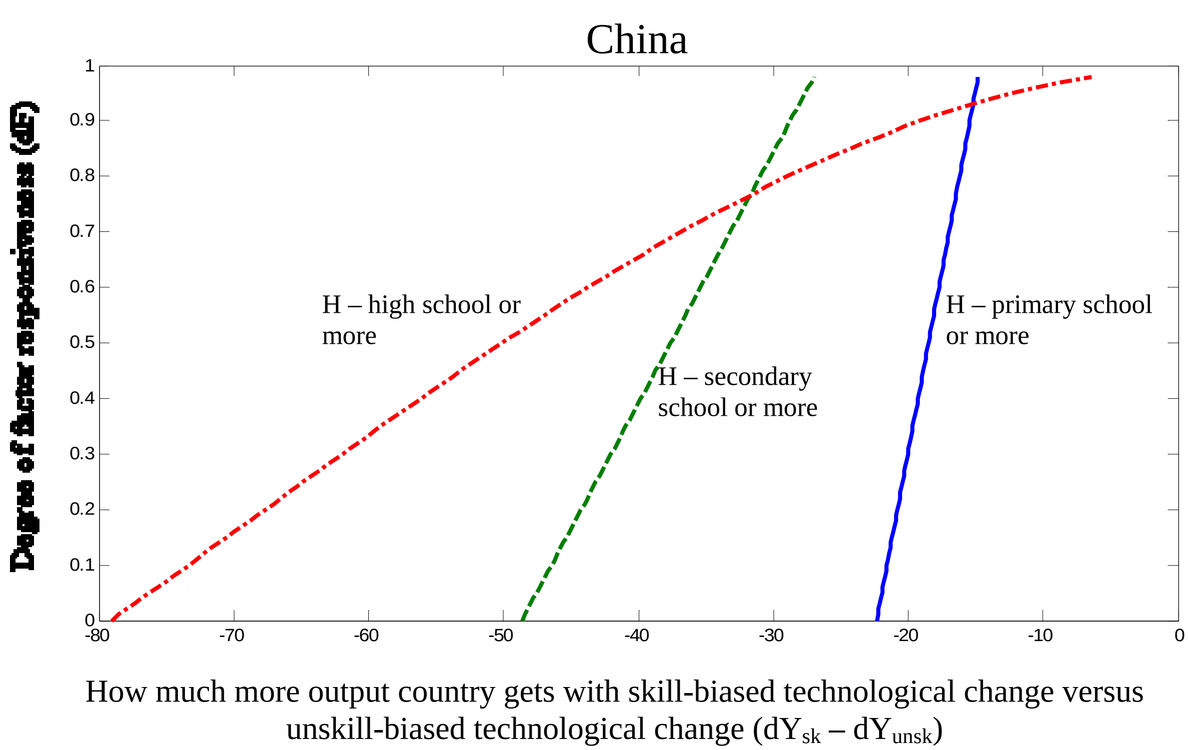


Figure 2 – Skilled vs. Unskilled Technological Growth When Factors Respond – Comparing GDP per Capita for Three Countries





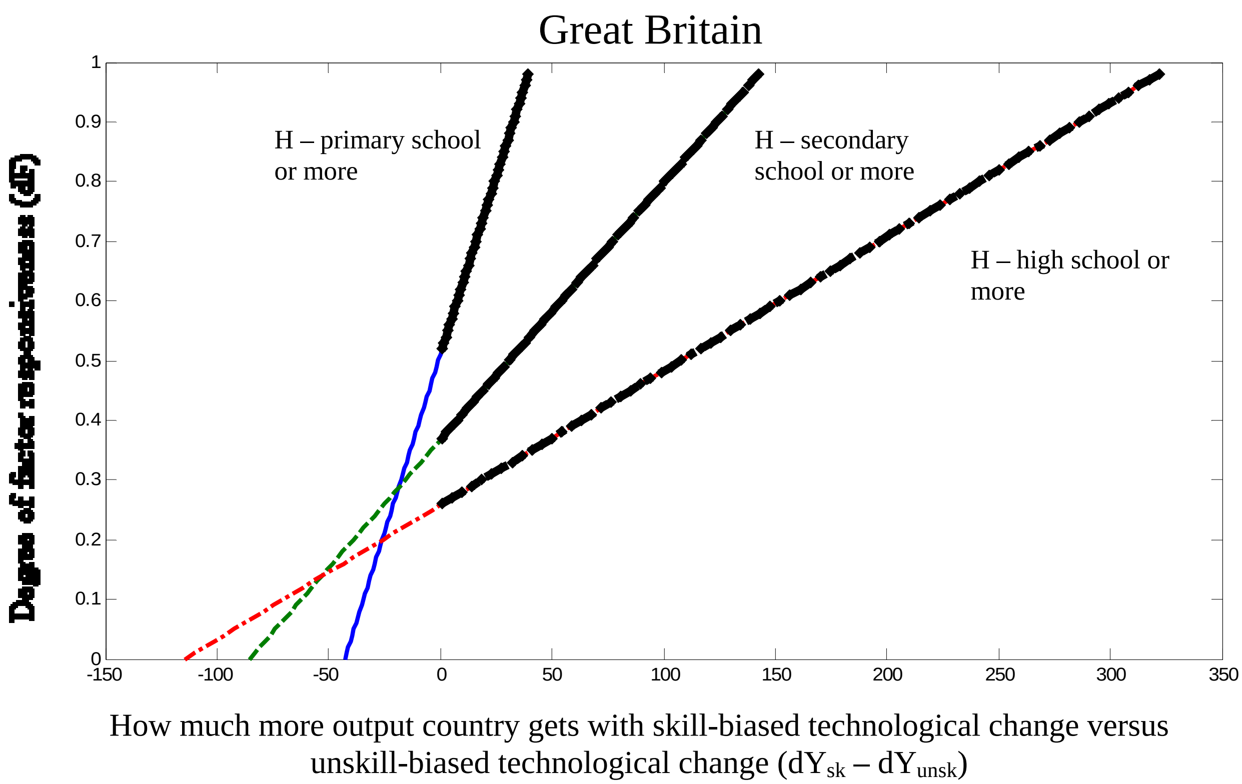

Great Britain
Degree of factor responsiveness (dF)
H – primary school or more
H – secondary school or more
H – high school or more
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-150
-100
-50
0
50
100
150
200
250
300
350
How much more output country gets with skill-biased technological change versus unskill-biased technological change ($dY_{sk} – dY_{unsk}$)

Figure 3 – How Much Must Factors Respond To Make Skill-Biased Technological Growth "Better?" – Cross Country Comparison

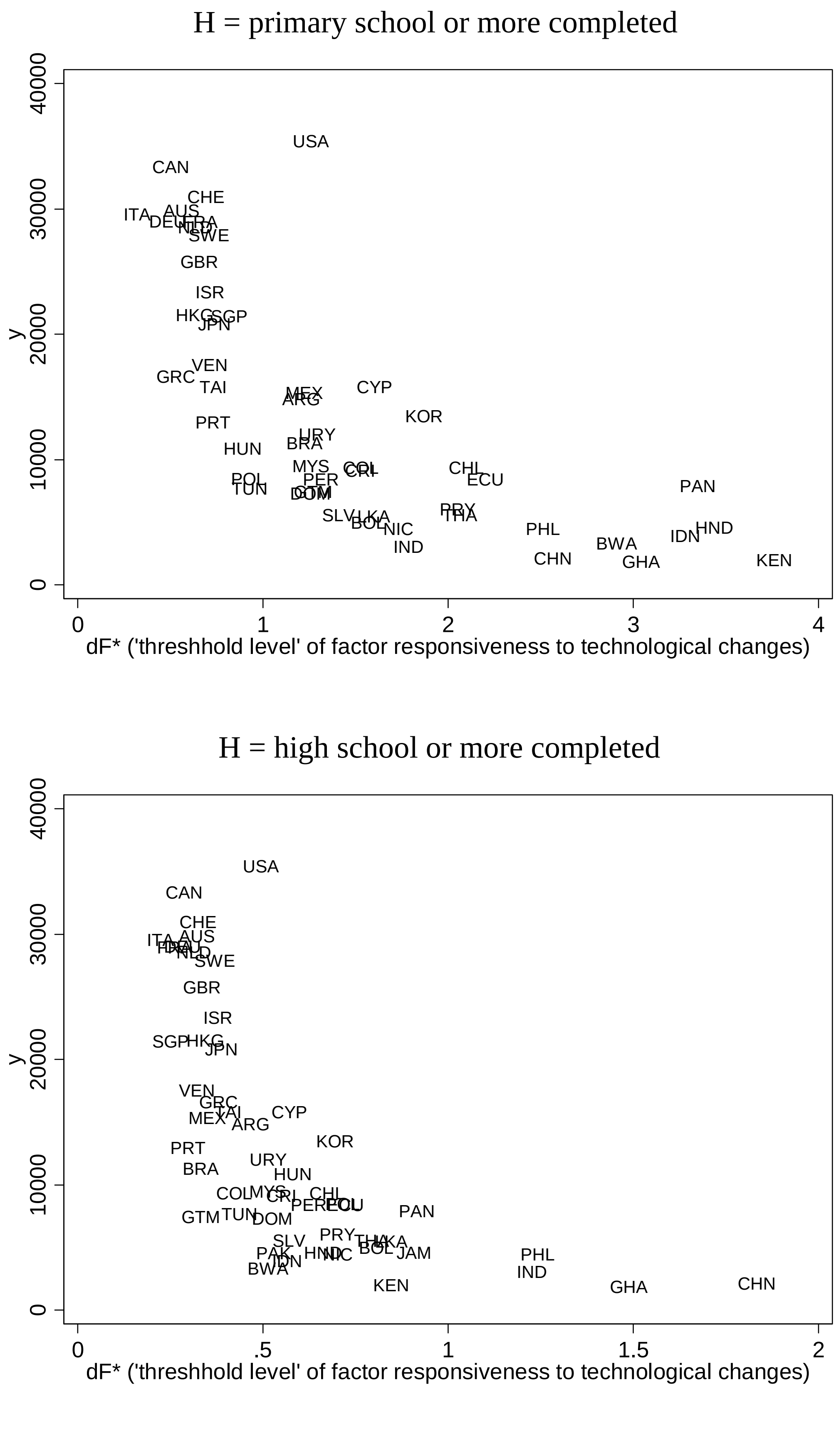
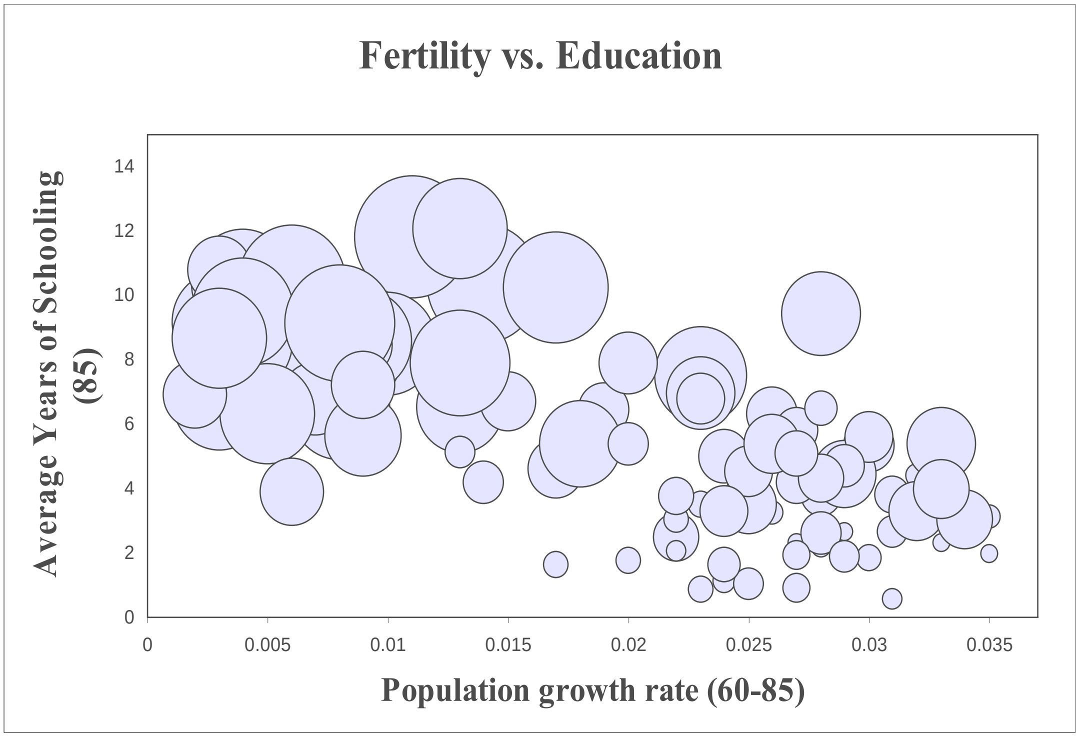
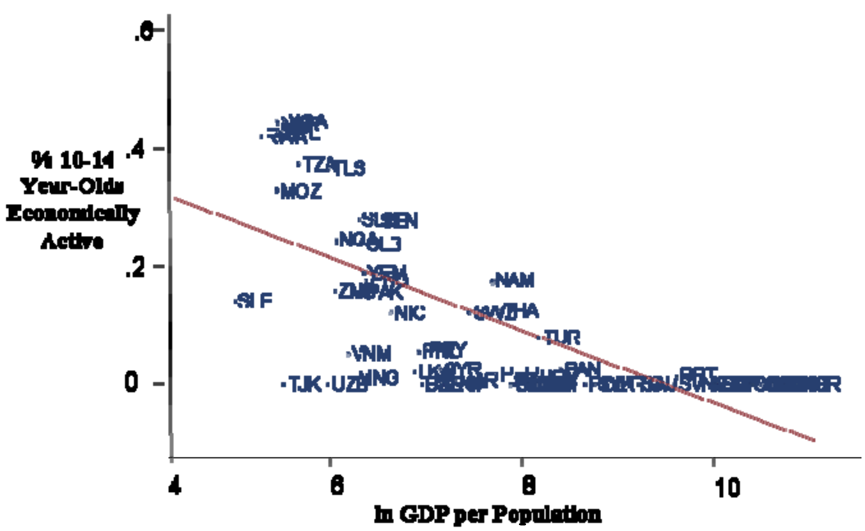


Figure 4 – Quality-Quantity Tradeoffs in Labor



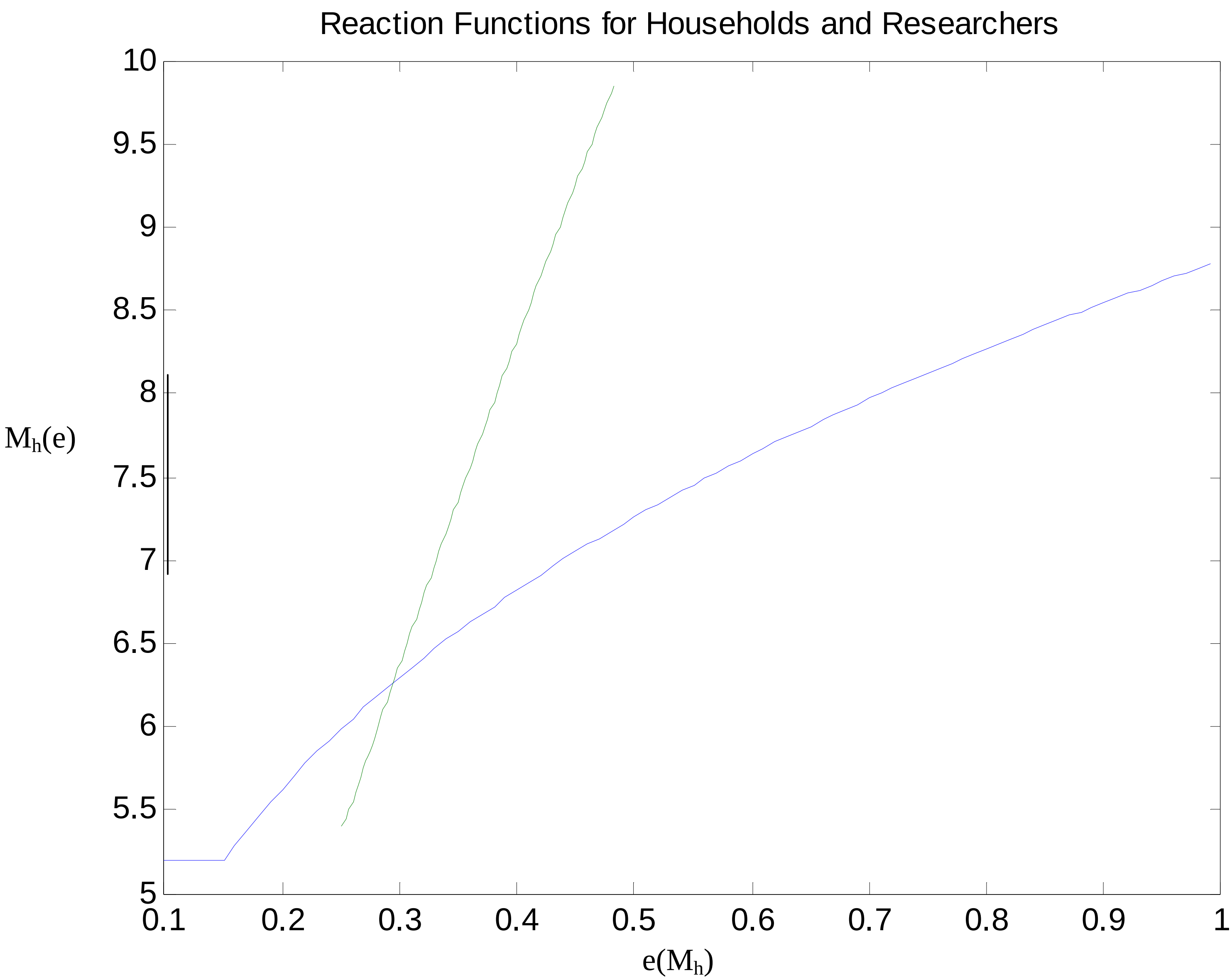
Source: Barro and Sala-i-Martin (2003). Observations are sized according to income per capita. The graph illustrates both that a negative relationship exists between cross-country measures of fertility and education rates, and that higher-income countries tend to be those with low population growth and high education rates.

Child-Labor Participation vs. Income



Source: International Labor Organization (2002)

Figure 5 – Reaction Curves



- The steeper line represents the fraction of educated young a parent would choose for a given technological parameter M_h. The flatter curve represents the skill-biased technical coefficient that would result from a given fraction of educated young. Note that for a very low level of education, no resources are devoted to skill-intensive research, in which case the skilled sector remains stagnant.

Figure 6 – Simulation of Economy with Exogenous Unbalanced Growth

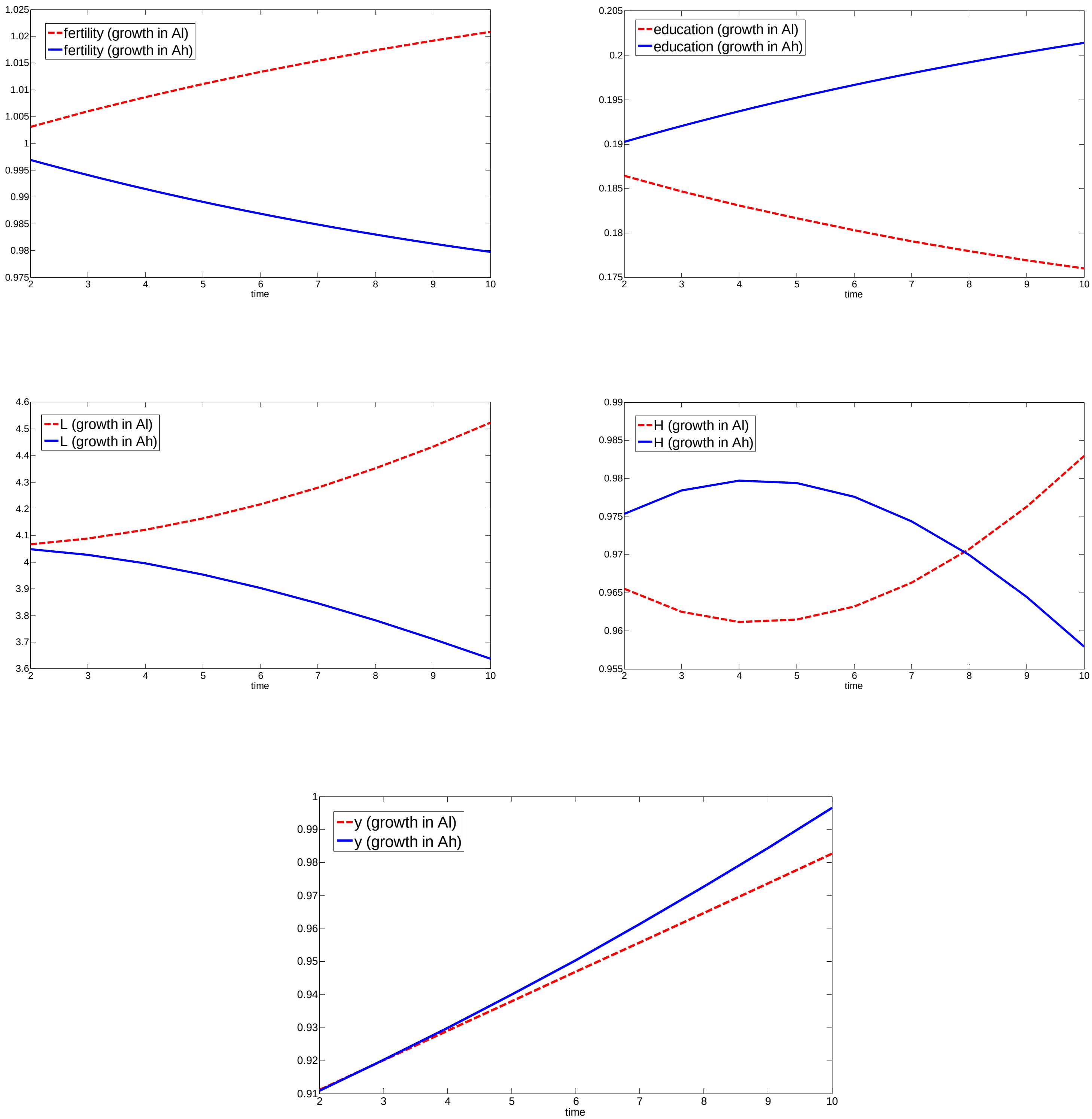


Figure 7 – Simulation of Economy with Endogenous Unbalanced Growth ($M_h \approx M_l$)

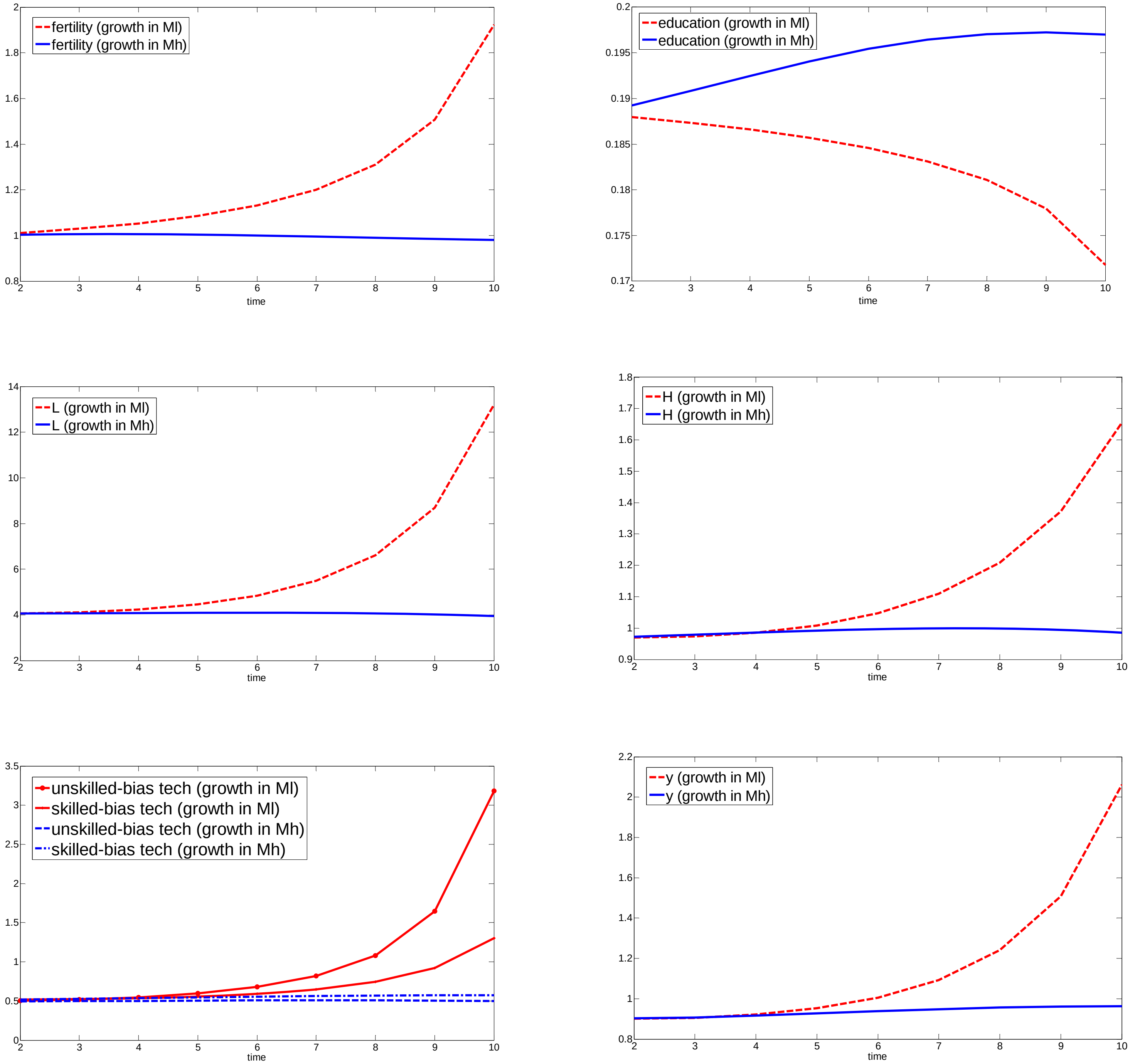


Figure 8 – Simulation of Economy with Endogenous Unbalanced Growth ($M_h > M_l$)

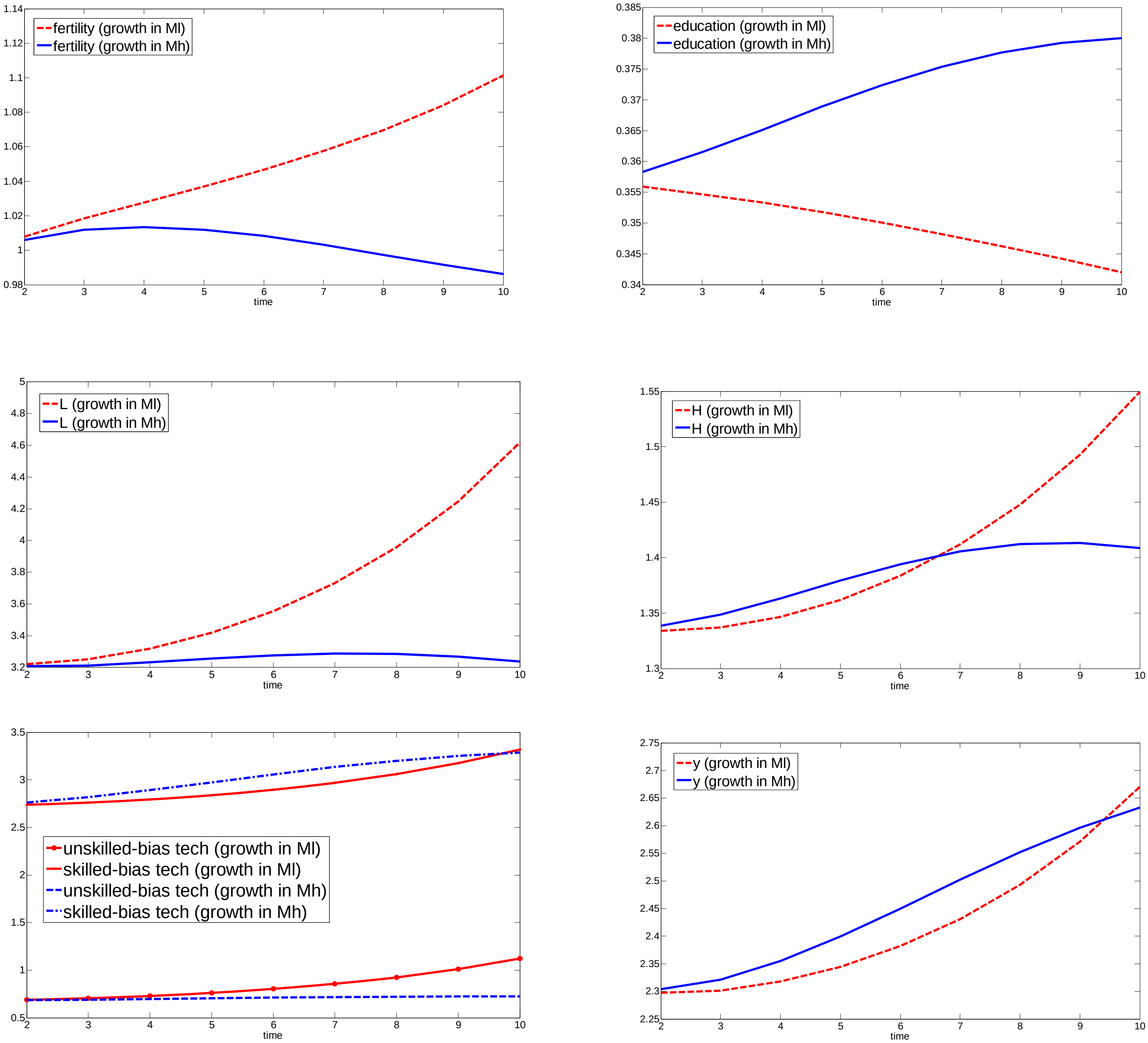


Table 1 – Cross-Country Values of A_l and A_h

	σ=0.33						σ=0.5					
	H=primary		H=secondary		H=college		H=primary		H=secondary		H=college	
Country	A_l	A_h	A_l	A_h	A_l	A_h	A_l	A_h	A_l	A_h	A_l	A_h
ARG	4.8	53.2	29.5	18.0	49.9	4.7	18.3	73.9	44.7	56.1	57.9	37.8
AUS	1.5	170.4	38.5	72.6	112.3	10.9	16.2	189.1	81.7	143.0	137.7	64.2
BOL	9.4	10.2	18.6	4.4	27.4	1.2	18.2	21.9	25.5	17.4	31.1	10.9
BWA	5.1	6.4	12.1	1.6	14.0	1.1	9.2	15.2	14.2	12.7	15.1	16.7
BRA	11.7	26.2	19.6	29.6	29.0	26.1	25.2	50.7	32.6	78.1	39.3	106.1
CAN	0.3	219.3	43.7	81.6	166.9	2.0	7.9	228.9	92.9	160.4	180.4	29.2
CHL	3.1	30.0	12.6	19.0	25.7	5.9	11.1	43.0	22.0	46.6	31.3	35.6
CHN	4.4	4.7	11.8	0.7	16.3	0.0	8.7	10.0	14.2	4.3	16.6	0.5
COL	6.2	28.3	17.5	19.3	30.1	7.9	16.7	46.7	27.8	55.2	36.3	50.8
CRI	7.4	23.9	17.6	16.3	26.7	10.3	18.5	41.2	28.5	45.2	34.9	47.3
CYP	1.7	60.8	12.2	44.6	34.9	12.9	10.3	74.3	27.0	84.6	45.2	60.5
DOM	13.5	13.9	25.9	6.8	34.2	3.4	25.4	31.2	35.0	27.9	40.1	25.0
ECU	2.8	28.1	9.6	22.1	17.8	14.2	10.0	39.8	18.5	47.9	25.1	52.1
SLV	14.3	7.6	26.3	2.3	31.8	0.8	23.3	20.8	31.6	14.7	34.6	11.5
FRA	6.1	123.7	51.2	48.4	102.2	9.5	29.4	159.0	84.5	129.7	118.6	74.8
GHA	5.5	2.6	11.7	0.2	13.2	0.0	8.8	7.2	12.7	2.4	13.6	1.1
GRC	6.5	91.1	71.4	12.7	107.9	2.1	30.4	118.1	99.4	48.1	121.3	21.4
GTM	12.9	14.6	24.3	9.0	31.4	6.1	23.6	33.9	32.3	38.9	36.6	47.1
HND	2.9	12.9	6.4	13.9	13.4	3.9	7.3	21.9	10.8	35.4	15.7	29.9
HKG	6.7	98.2	44.8	37.1	108.8	2.1	30.2	128.7	77.2	92.9	119.3	26.2
HUN	4.9	49.3	45.1	5.4	61.2	1.4	19.7	67.1	58.7	25.1	68.4	14.3
IND	10.7	3.5	18.0	0.9	22.7	0.1	16.6	10.6	21.4	6.0	23.9	2.5
IDN	2.4	13.1	10.3	3.8			6.5	21.3	13.3	17.7		
ISR	4.4	102.0	25.1	63.2	82.0	9.3	23.8	127.5	56.2	118.6	100.6	54.2
ITA	34.3	109.4	157.3	13.6	231.6	0.7	95.1	176.6	201.2	66.9	243.6	16.9
JAM	0.1	15.7	1.2	28.6	3.8	24.8	1.0	18.3	2.9	51.2	5.2	100.9
JPN	2.5	105.6	37.5	35.1	79.6	7.2	17.5	125.4	65.9	85.6	95.7	46.5
KEN	4.4	3.0	9.2	0.5	10.1	0.3	7.1	8.3	10.2	4.9	10.7	6.4
MYS	5.6	34.3	26.7	9.9	49.2	0.3	17.5	51.8	37.9	35.6	51.4	8.5
MEX	6.8	48.7	21.6	36.5	39.1	17.9	21.0	74.0	37.2	91.6	49.5	91.6
NLD	2.1	150.0	52.1	43.1	105.8	7.9	17.9	171.6	87.5	111.9	124.1	58.4
NIC	10.8	6.5	17.1	3.9	21.1	2.7	18.1	17.2	22.6	17.0	25.2	18.1
PAK	15.1	5.9	24.4	2.2	31.4	0.5	23.2	17.7	29.5	14.2	33.5	8.4
PAN	1.1	28.1	5.6	24.8	14.7	10.5	5.4	35.7	12.2	47.5	19.7	44.2
PRY	6.1	14.4	16.1	6.4	23.3	2.4	13.9	26.7	22.4	24.0	26.9	19.8
PER	7.2	25.5	19.6	13.7	32.7	4.9	19.2	42.2	31.5	38.2	40.3	28.0
PHL	1.6	18.8	8.2	9.0	14.9	3.5	6.4	25.5	14.4	21.7	19.3	16.7
POL	1.3	53.1	37.3	4.7	54.9	0.6	9.7	62.6	49.5	20.5	59.9	7.9
PRT	14.8	41.9	50.5	10.2	71.1	2.0	35.9	73.6	65.8	47.3	77.4	27.0
KOR	0.5	61.0	8.4	42.8	32.0	7.9	5.1	68.2	20.7	74.8	39.9	42.6
SGP	9.4	75.6	42.8	33.5	75.0	8.2	30.6	112.2	64.5	105.5	85.3	74.2
LKA	4.0	20.7	21.7	3.3	34.8	0.0	12.2	31.7	28.2	15.3	35.5	1.7
SWE	2.5	133.5	20.4	93.6	95.6	8.8	19.1	155.6	54.0	155.5	114.9	57.1
CHE	1.4	157.6	22.1	103.0	112.6	5.1	14.5	176.0	56.9	174.2	127.2	48.4
TAI	4.8	74.8	35.1	26.8	79.7	2.2	22.4	97.5	59.6	68.7	89.0	23.5
THA	6.4	12.6	18.2	3.6	19.7	4.1	14.0	24.2	23.5	17.1	24.3	23.6
TUN	22.3	11.1	35.4	5.3	50.6	0.8	36.4	30.4	45.5	25.7	54.6	12.6
GBR	4.9	117.1	56.7	30.7	105.5	5.3	26.6	146.0	89.0	89.6	121.0	44.8
USA	0.0	146.7	3.4	162.5	48.6	37.7	1.8	149.4	19.3	201.4	71.2	126.0
URY	5.2	41.6	26.3	14.3	44.2	3.9	17.8	60.3	39.5	45.4	51.3	30.9
VEN	17.0	50.0	44.6	25.0	74.3	7.6	42.7	86.0	68.5	75.6	88.3	52.2
DEU	12.4	122.1	112.6	13.1	163.5	1.4	49.0	167.4	145.2	62.9	175.1	23.9