# QUANTITATIVE AND SYMBOLIC REASONING CENTER
## HAMILTON COLLEGE

# Mathematical Review: Computation

Ratios and Proportions

A **ratio** is a fractional representation of two measured quantities.

Ex. The ratio of 200 minutes to two hours is: $200/(2\cdot 60) = 5/3$

A **proportion** is a statement that equates two ratios.

Ex. Sarah is driving home for spring break. In her Saturn Ion she can travel 400 miles on 20 gallons of gas. At this rate, how many more gallons will she need for a 900 mile trip?

Let G represent the number of gallons used. The proportion would look like:

$$\frac{20 \text{ gallons}}{G \text{ gallons}} = \frac{400 \text{ miles}}{900 \text{ miles}}$$

$G = (20g\cdot 900mi) = (Gg\cdot 400mi) \rightarrow G = 45$ gallons

**Part-whole Percentage Problems**

The basic equation for determining a percentage or part is:

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percentage}}{100}$$

**Ex. 1** Hamilton accepted more students than usual this year, resulting in chaos and tears at the housing lottery. By the end of the lottery only 467 rising sophomores had housing. If there are 502 rising sophomores in the class, what percentage of students are *without* housing/will be living in tree houses in the glen?

$$\frac{35 \text{ (part)}}{502 \text{ (whole)}} = \frac{p \text{ (percentage)}}{100}$$

$$3500 = 502p$$

$$p = 6.97\% \rightarrow \sim 7\%$$

**Practice:**

I. It was recently announced that this year's Great Name's speaker is to be Richard Simmons. If 38% of students wanted Simmons, 32% wanted Bill Murry, 17% wanted Bono, and 13% wanted Tony Blair, how many students voted for Bill Murry? Assume that 1,486 students voted.

II. Luke bought his Real Analysis textbook used at the bookstore. If he paid $69.64, and the price for the same book new is $92.85, what percentage did he save?

Nutrition Exercise

In order to follow any of the current healthy diets, you need to be able to *calculate* the approximate fat, protein, carbohydrates, and fiber content of the foods you eat every day.

*Determining your daily caloric requirements*

(Source: *The Complete Book* by Kurt Brungardt)

a. Multiply your current weight by the appropriate factor for your gender to create estimate baseline caloric needs.

Men: _____ lbs. x 11.0 = _____

Women: _____ lbs. x 10.0 = _____

b. Multiply your current weight by the approximate *activity factor* to estimate calories needed for each activity.

| | Sedentary | Light | Moderate | Heavy |
|---|---|---|---|---|
| Men: | 3.2 | 6 | 7.2 | 10.5 |
| Women: | 3 | 5 | 6 | 9 |

_____ lbs. x _____ = _____

*Sedentary:* little exercise/sit-down job
*Light:* some exercise/on your feet most of the time
*Moderate:* exercise 3-5 times a week/moderately active lifestyle
*Heavy:* exercise 5 or more times a week/very active job or lifestyle (eg. construction work)

c. Add together the calories from parts a and b. Then, based on your calculation of your desirable weight range, adjust: add 500 calories if you are currently under the desirable range, or subtract 200 if you are currently over the desired range; add 0 if you are in the desirable range.

GET NEW WEIGHT RANGE CHART!!!!

_____ + _____ ± _____ = _____ Total daily caloric need
a b (adjust)

*Determining your daily needs for carbohydrates, fats, and protein*

Note: These calculations assume you need 60% of your calories from carbohydrates, 25% from fats, and 15% from proteins.

a. Multiply your total daily caloric need from above by 60%, 25%, and 5% respectively to get the number of calories needed from carbohydrates, fats, and proteins.

_____ x .60 = _____ needed calories from carbohydrates
_____ x .25 = _____ needed calories from fats
_____ x .15 = _____ needed calories from proteins

b. Finally, we use the fact that a gram of carbohydrate contains 4 calories, a gram of fat contains 9 calories, and a gram of protein contains 4 calories to convert the caloric needs to needs expressed in grams:

_____ (carb cal.) / 4 = _____ needed carbohydrates in grams
_____ (fat cal.) / 9 = _____ needed fat in grams
_____ (protein cal.) / 4 = _____ needed protein in grams

**Practice:**

I. Chris decides to follow the 60% carb, 25% fat, and 15% protein diet. If he requires a total of 3450 calories a day, how many grams of carbohydrates, fats, and proteins is Chris allotted per day?

II. Chris went to the fitness center after his morning class and spent 30 minutes on the stair-climber and then played racquetball for 45 minutes. How many calories did Chris burn off? Assume that an hour on the stair-climber at moderately high resistance or an hour of racquetball will burn off roughly 700 calories.

III. Heavy cream is 40% fat (by weight), regular milk is 3.3% fat, 2% and 1% milk are, obviously 2% and 1% fat respectively, and skim milk is almost 0% fat. If a cup of each weighs roughly 245 grams, how many grams of fat are in a cup of each?

IV. Compute the cost of each cereal in dollars per gram, and dollar per ounce:

| | Weight | Weight | Servings | Price |
|---|---|---|---|---|
| Cheerios | 10 oz | 1 cup = 30g | 9 | $2.49 |
| Corn Flakes | 18 oz | 1 cup = 28g | 18 | $2.89 |
| Raisin Bran | 15 oz | 1 cup = 61g | 7 | $2.59 |

Personal Finance Exercise

Almost everyone has a credit card now, and people use them for everything from buying groceries to booking hotel rooms to placing down payments on cars. So, you should be very aware of how the periodic interest can work against you; each month you are assessed a monthly finance charge that is determined by calculating the average daily balance in your account and multiplying it by a monthly (or sometimes daily) periodic interest rate. This monthly finance charge is then added to your account's total amount due, like any other purchase, and then becomes part of your average daily balance when it comes time to compute next month's finance charge (unless, of course, you pay off the balance before the payment due date). In the event you do not pay off the entire balance in your account each month, you are paying *interest on interest.*

Conversely, if you have a savings account with a balance of $6000 and have an annual interest rate of 1.17%, on the last day of each month the interest is computed and added to your account. In this case, at the end of the month your balance would grow by $5.85.

$$6000 \times (0.0117 / 12) = 5.85$$

Now, assuming that you made no other deposits or withdrawals for the next month, the bank would add more interest to your account:

$$6005.85 \times (0.0117 / 12) = 5.8557 = 5.86$$

By the beginning of the third month, the balance in your savings account is $6011.71!

| Date | Balance | Interest |
|---|---|---|
| 1-Jan | 6000 | 0 |
| 31-Jan | 6000 | 5.85 |
| 1-Feb | 6005.85 | 0 |
| 28-Feb | 6005.85 | 5.86 |
| 1-Mar | 6011.71 | 0 |
| 31-Mar | | |
| 1-Apr | | |
| 30-Apr | | |
| 1-May | | |
| 31-May | | |
| 1-Jun | | |
| 30-Jun | | |
| 1-Jul | | |
| 31-Jul | | |
| 1-Aug | | |
| 31-Aug | | |
| 1-Sep | | |
| 30-Sep | | |
| 1-Oct | | |
| 31-Oct | | |
| 1-Nov | | |
| 30-Nov | | |
| 1-Dec | | |
| 31-Dec | | |

Fill in the remaining entries, assuming no additional deposits or withdrawals.

**Practice:**

I. Chelsea has not used her credit card all month and so decides to go on a shopping spree at the mall using her card. Looking at the information below, and assuming that Chelsea does not make any further purchases and that she did not have an outstanding balance, determine the finance charge for that month and the new balance. (Hint: to determine the monthly charge, remember to divide the annual percentage by 12!)

| | |
|---|---|
| Macy's | $52.68 |
| Bath and Body Works | $17.55 |
| Target | $14.26 |
| Hot Topic | $54.85 |
| FYE | $43.89 |
| Total | $183.23 |
| Annual Percentage rate: | 17.650% |

II. Suppose you have a Visa account balance of $3000; that is, you owe $3000. Furthermore, suppose the annual percentage rate is 17.65%. Considering your only on-campus job is working at the mail center, you are very concerned about how to pay this bill off! So, after cutting the card into eight different pieces, you vow never to charge that much ever again. You then determine that the largest monthly payment you can afford right now is $50 per month.

a. Produce a month-by-month balance chart similar to the one on page 5 for the upcoming year. Assume that your payment is received by Visa on the last day of each month and that the monthly finance charge is added to the total balance on that date as well. How much do you owe on the account after a year of paying $50 a month? (Hint: you can use Excel for this problem)

b. Suppose that the interest rate was 19.15%. Then how much would you owe at the end of a year?

c. What would your monthly payment need to be to pay off the $3000 balance in 2 years given an annual percentage interest rate of 17.65%? What would be the total amount you paid Visa in those two years?

III. Remember the savings account example from page 5? We described the balance for the first month as: 6000 x (0.0117 / 12).

Note that you can find the balance for the first month by multiplying the first month's balance, 6000, by 1.000975. What function does the "1" play in the computation? What function does the ".000975" play in the computation?

The "1" retains the previous months balance and the ".000975" adds that month's interest to that balance.

Additionally, exponential functions are an easy way of determining interest on a savings account. Generally, for an amount $A_0$ invested at an interest rate $r$, after $t$ years the account is worth $A_0(1 + r)^t$. However, interest is usually compounded more frequently then just at the end of the year. So, for $n$ times a year the interest rate is $r/n$ and there are $nt$ compounding periods in $t$ years:

$$A_0(1 + {}^{r}/_{n})^{nt}$$

For instance, if \$3,000 is invested at 2% interest for three years it will be worth:

$\$3000(1 + (.02)/1)^{(1)3} = 3000\ (1.02)^3 = \$3{,}183.62$ *(annual compounding)*

$\$3000(1+ (.02)/12)^{(12)(3)} = 3000(1.002)^{36} = \$3{,}223.73$ *(monthly compounding)*

a. Generalize your findings. Suppose you put \$D into a special savings account that receives r% interest annually and is compounded monthly. Assuming you make no further deposits or withdrawals, write an expression that describes the balance in the account…

…after 1 year _____

…after 5 years _____

…after 10 years _____

…after 20 years _____

b. Let's suppose your Aunt Betty White gives you \$10,000 as a gift for graduating from Hamilton. Assume that you are 22 when you receive this gift. You put it away in a CD (certified deposit) guaranteeing 8% annual interest as long as you make no deposits or withdrawals. You leave the money in the account until you retire at 72. What is the value of the account at that time?