

QUANTITATIVE & SYMBOLIC REASONING CENTER HAMILTON COLLEGE



Mathematical Review:
Graphs and Charts

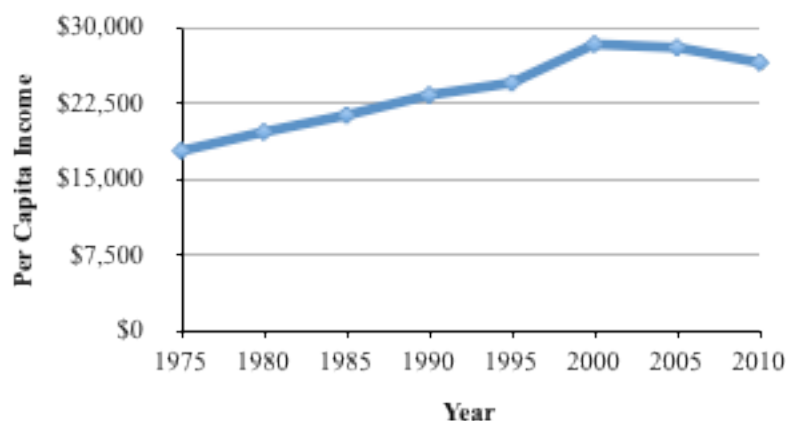
Graphing One Variable: Time-Series Graphs and Bar Charts

Graphing is a visual means to represent events and formulae. In economics, graphs are particularly useful in showing how variables change, both with respect to time and with respect to other variables. Graphs showing changes in variables with respect to time are called **Time-Series Graphs**. In these graphs, time is usually plotted on the x-axis, also called the horizontal axis, while the variable of interest (e.g. Income), is plotted on the y-axis, also called the vertical axis. Because Time-Series Graphs show the changes in something over time, it is necessary to plot the time successively (e.g. 2000, 2005, 2010). This way, the graph can also be read from left to right. Time-Series graphs are useful in determining what kind of change, if any, has occurred to an economic variable over time.

The table below shows the numbers that we will put into a time-series graph. The left hand column shows the year, and the right hand column shows the average income per person in the USA, also known as *per capita income*. These data represent how much a person's income is worth had they received it in the year 2010.

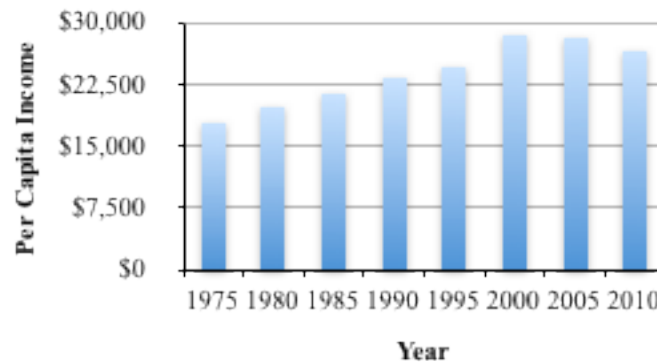
<u>Year</u>	<u>Per Capita Income</u>
1975	\$17,753
1980	\$19,618
1985	\$21,282
1990	\$23,266
1995	\$24,472
2000	\$28,293
2005	\$27,961
2010	\$26,487

The graph below puts these data into a more easily understandable fashion, and allows us to see trends by connecting the points. In the case of per capita income, the graph shows that income grew until the year 2000, after which it started to decline, though the rate of change



varied in each period.

These data can also be represented in a **bar chart**. A bar chart displays data in columns, with the height of each column representing the value of the economic variable (e.g. income). Below is a bar chart of Per Capita Income data.



Problems

One of the most frequently discussed problems by economists and policymakers is the growing *federal debt*. This number represents how much money the federal government owes to its various lenders. In studying the federal debt over time, one must also note the growth of the US economy over time. The most common measure of the size of the US economy is the *gross domestic product* (GDP), which is the value of all goods produced by a nation in a given year. Below is a table comparing federal debt and GDP by year (Debt and GDP are both expressed in millions of dollars).

<u>Year</u>	<u>Federal Debt</u>	<u>GDP</u>
1975	\$541,925	\$1,560,200
1980	\$909,041	\$2,724,200
1985	\$1,817,423	\$4,146,300
1990	\$3,206,290	\$5,734,500
1995	\$4,920,586	\$7,341,100
2000	\$5,628,700	\$9,821,000
2005	\$7,905,300	\$12,428,600
2010	\$13,528,807	\$14,359,700

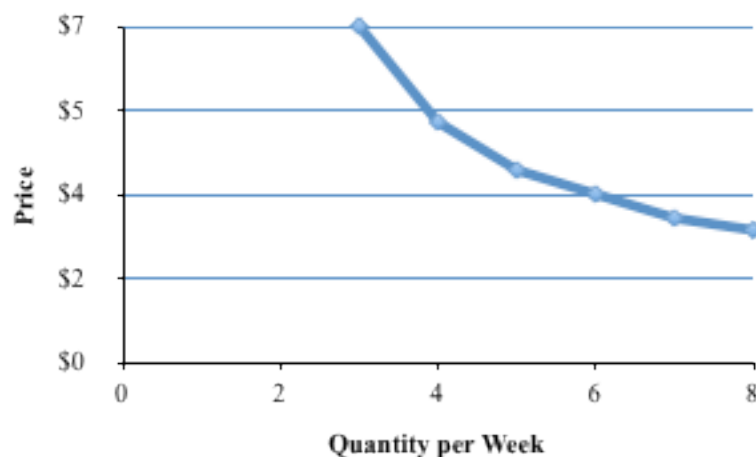
1. Create a time series graph showing the change in federal debt over time.
2. Show the same data on a bar graph.
3. Using these figures, compute *federal debt as a percentage of GDP* and display the data on a time-series graph.
4. What does a comparison between the two time-series graphs show?

Graphing Two Variables: Scatterplots and Relationships

Sometimes graphing with only one variable isn't enough. In economics, it is often necessary to compare an economic variable not just with time, but with other economic variables, in order to see if a relationship exists between them. These graphs are frequently called **scatterplots**. One of the most common examples of this kind of analysis is the *relationship between price and quantity*. Below is a table comparing the price of apples to the quantity of apples that an average consumer will purchase at that price in a given week.

<u>Price</u>	<u>Quantity per Week</u>
\$7	3
\$5	4
\$4	5
\$3.50	6
\$3	7
\$2.75	8

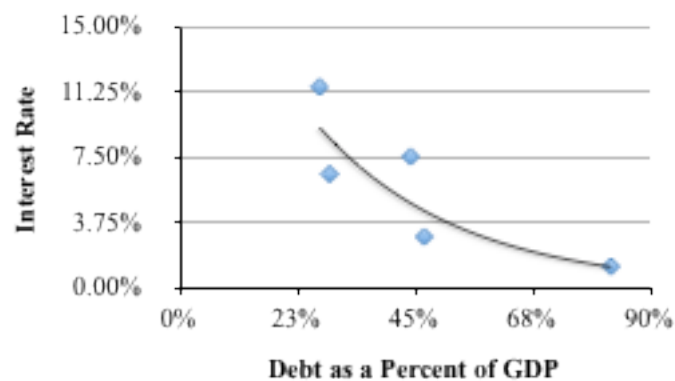
These data can be placed on a scatterplot, as shown below. In economics, price is plotted on the y-axis and quantity is plotted on the x-axis. The scatterplot allows us to draw a curve through the points, and see that as the price of an apple increases, consumers purchase fewer of them.



Some cases are not as clear, however. The table on the next page shows the US debt as a percent of GDP and the *interest rate* the government must pay on its debt over time.

<u>Year</u>	<u>Debt as Percent of GDP</u>	<u>Interest Rate</u>
1950	82.5%	1.2%
1960	46.8%	2.9%
1970	28.7%	6.5%
1980	26.8%	11.5%
1990	44.2%	7.5%

The scatterplot below shows a correlation between the interest rate and the debt percentage, meaning that as the interest rate increases, the debt percentage tends to decrease. This doesn't mean that the change in interest rate causes the change in debt percentage; there could be other factors this scatterplot doesn't consider. Recognizing that correlation doesn't imply causation is very important in analyzing any graphs. This principle can also be applied to the apple chart above.



Problems

The table below shows the data on the *risk* and *expected rate of return* (as a percentage) of various hypothetical investments.

<u>Asset</u>	<u>Expected Rate of Return</u>	<u>Risk</u>
Bank Deposit	3	0
U.S. Treasury Bills	4	3
TopCorp Bonds	9	10
ABC Stock	11	24
XYZ Stock	13	24
RiskyCorp Stock	16	39

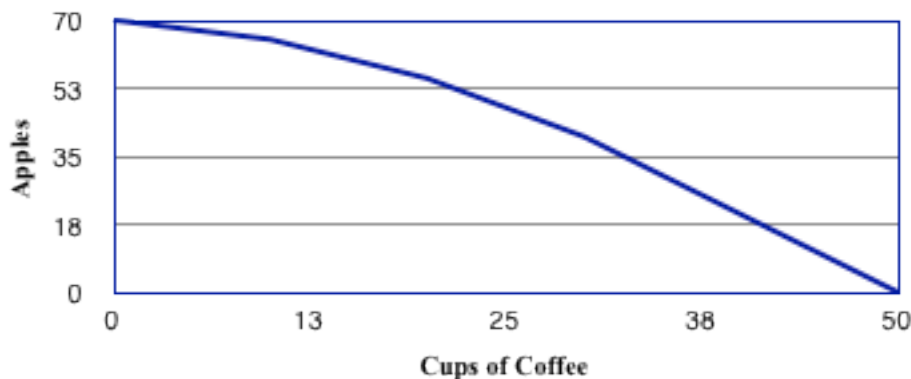
1. Graph these data on a scatterplot.
2. What does the graph suggest about the relationship between expected rate of return and risk?

Slopes of Lines and Curves

The **slope of a line** is the rate at which a line changes between points. The slope is most commonly measured by dividing the change in the y-axis coordinates of 2 points over the change in their x-axis coordinates. In a linear equation written in the slope intercept form, $y = mx + b$, m represents the slope, and b represents the vertical intercept. The **slope of a curve**, however, changes and must be measured at a particular point. The slope at a particular point is defined as the slope of a tangent line to the curve at that point.

Problems

1. A previous example of a scatterplot examined the relationship between the quantity of apples purchased in a given week and their price. This relationship is called a *demand curve*. A central hypothesis to demand is that as the price of a good falls, the quantity demanded will increase, holding all other possible factors (e.g. income) constant.
 - a. Suppose the demand for pizzas is represented by the equation $P = 15 - 0.01Q$, where Q is the quantity of pizza demanded and P is the price per pizza.
 - i. How many pizzas will be demanded at a price of \$5? Of \$10? Of \$15?
 - ii. Rather than graphing the demand curve, placing Q on the y-axis and P on the x-axis, economists traditionally graph the inverse demand curve, analyzing quantity as it impacts price. To do this, isolate P in the regular demand equation. In this example, the inverse demand curve is $Q = 1500 - 100P$. Graph this curve.
 - iii. Find the slope and vertical intercept of the inverse demand curve.
 - iv. Explain what the slope of the inverse demand curve means in terms of the relationship between price and quantity.
 - b. Suppose the demand curve changes to $P = 15 - 0.005Q$.
 - i. Find the equation of the inverse demand curve.
 - ii. How does this inverse demand curve compare to the inverse demand curve in part a?
 - iii. Write an equation for a demand curve, and its inverse, with the same slope as the demand curve in part a but with a vertical intercept of (0, 350)
 - c. Suppose the demand curve changes to $P = 15 - 0.01Q$.
 - i. Find the equation of the inverse demand curve.
 - ii. How does this inverse demand curve compare to the inverse demand curve in part a?
 - iii. Write an equation for a demand curve, and its inverse, with the same vertical intercept as the demand curve in part a, but with a slope of 15.
2. Economists use a graph called the *production possibility frontier* (PPF) to discuss the use of scarce resources. This graph shows all possible combinations of goods that can be produced if resources are used efficiently. A hypothetical PPF for apples and cups of coffee is shown on the below



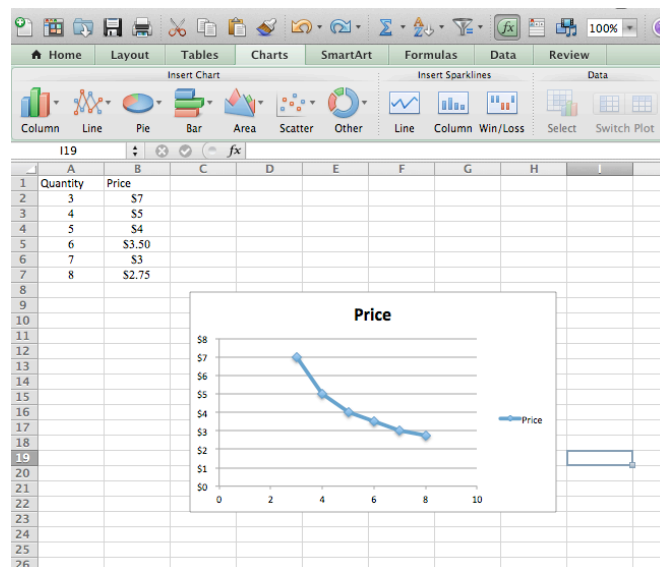
- a. Describe the slope of the PPF.
- b. What does the shape of the PPF tell us about the tradeoff between apples and cups of coffee?

Developing Graphs in Microsoft Excel

Microsoft Excel is a spreadsheet program that is excellent at making graphs. To begin, enter your X-axis and Y-axis values as shown in the screenshot below.

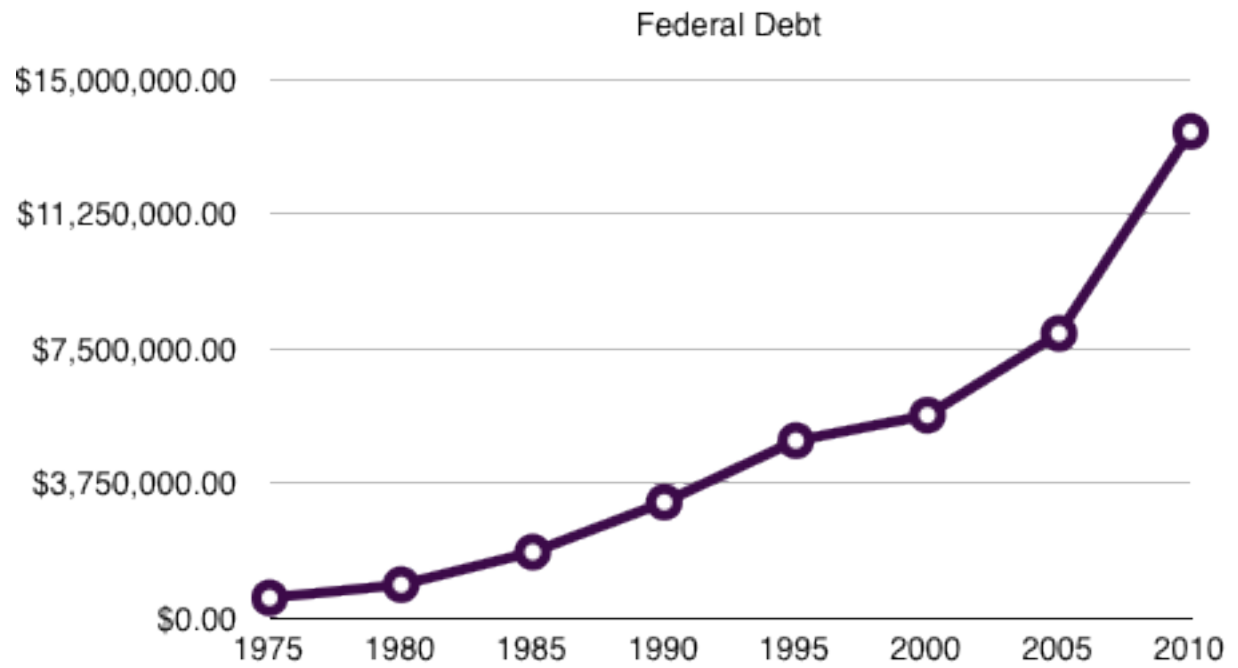
	A	B	C
1	Quantity	Price	
2		3	\$7
3		4	\$5
4		5	\$4
5		6	\$3.50
6		7	\$3
7		8	\$2.75
8			
9			
10			

Once you've entered your values, go to the charts tab and choose the kind of chart you would like to use. If you only have one set of X and Y-axis values, Excel will automatically turn them into a chart.

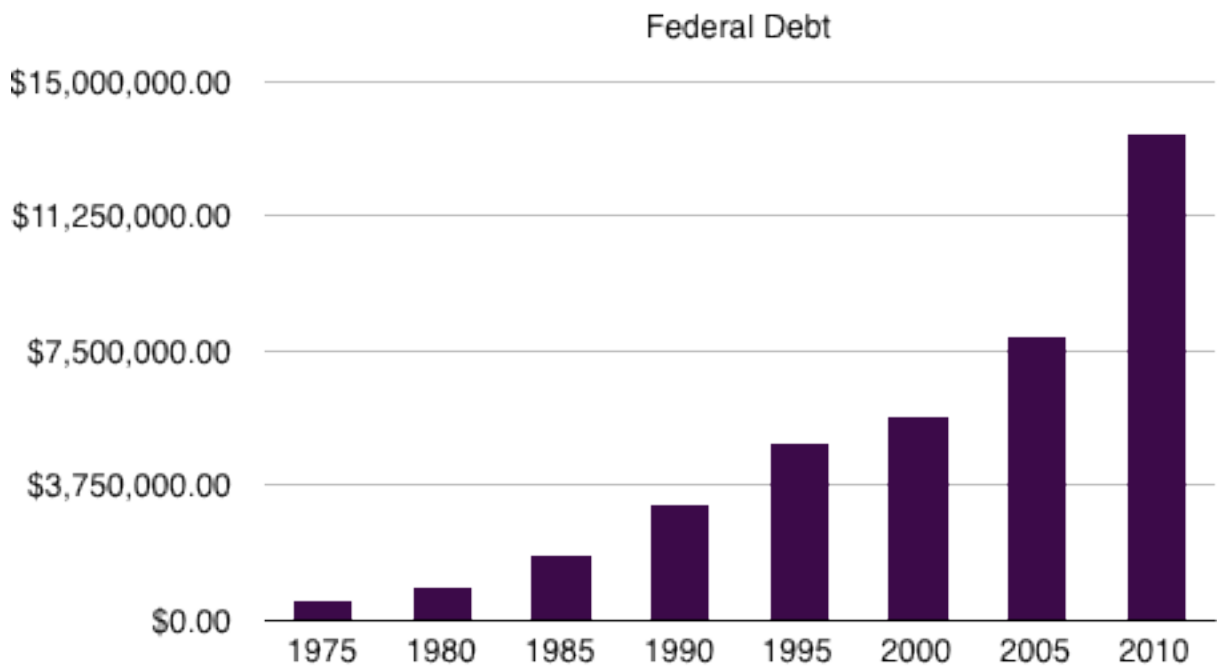


From here, you can edit the chart to change the title, add axis titles, labels, legends, and more! Just click on the part of the chart you want to edit.

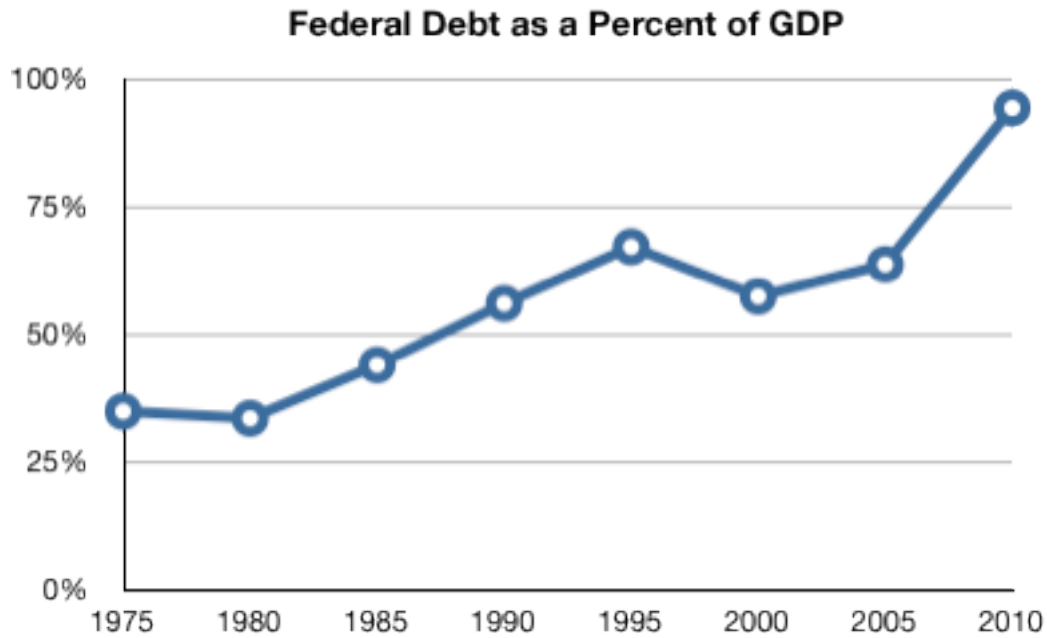
Answer Key: Graphing One Variable



1.

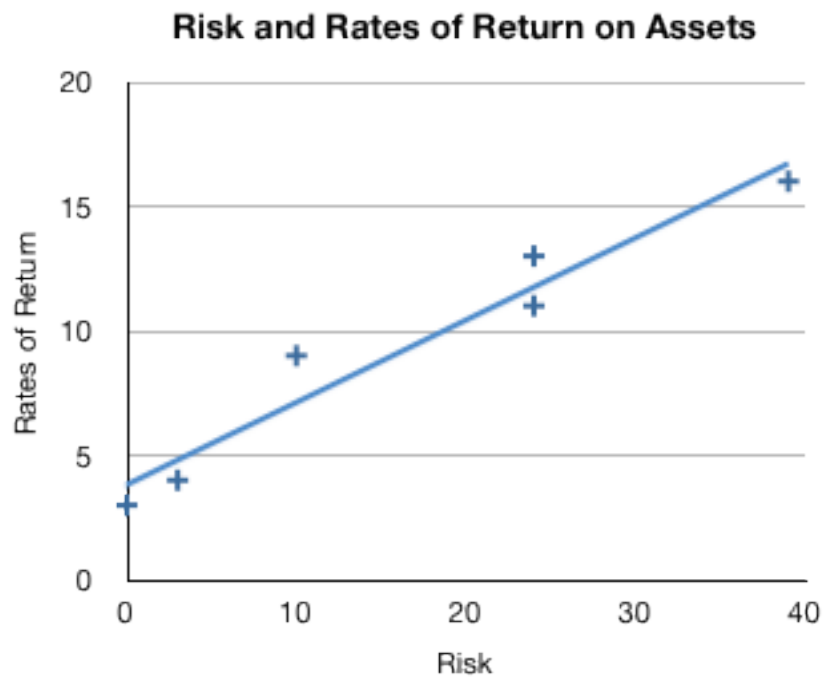


2.



- 3.
4. A comparison between the graphs shows that while Federal Debt has consistently grown over time, its growth as a percent of GDP hasn't been so linear.

Answer Key: Graphing Two Variables



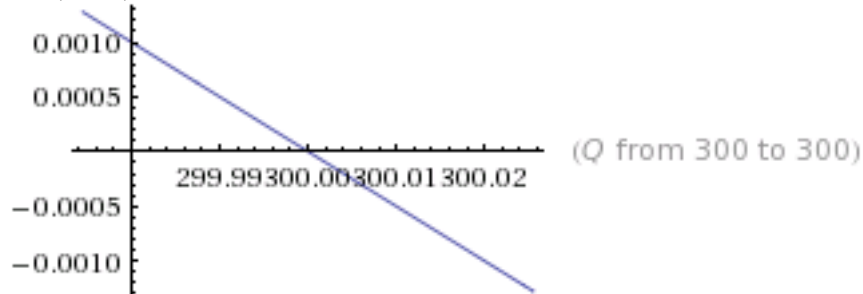
- 1.
2. The graph suggests a strong correlation, but only a correlation between risk and rates of return.

Answer Key: Slopes of Lines and Curves

1.

a.

i. 200, 100, 0



ii.

iii. $1/20$, 15

iv. This slope shows that for each additional pizza, the price demanded would drop by $1/20$.

b.

v. $P=20-(Q/20)$

vi. The y-intercept is higher.

vii. $Q=350-20P$; $P=(350/20)-(Q/20)$

c.

viii. $P=30-(Q/10)$

ix. The slope is steeper and the Y-Intercept is higher.

x. $Q=300-15P$; $P=20-(Q/15)$

2.

a. The slope of the PPF is non-linear and slopes downward, and is concave downward.

b. The shape of the PPF shows that as you produce more cups of coffee, you will produce fewer apples, and each additional cup of coffee will cost more and more apples.